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Optimal Levels of Reserves and Hedging Sudden Stops Recessions for Egypt: A Stochastic Control Approach

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This paper discusses the issue of optimal international reserves in Egypt during the period 1977-2007. We derive, using stochastic control, the optimal level of reserves to maximize the welfare of the society as measured by the utility of the consumption. The paper also presents a new approach to manage international reserves. It is proposed that part of the reserves will be held as risky assets. The risky assets are sensitive to the volatility index (VIX) of the Chicago Board Options Exchange (CBOE). The analysis reveals that the actual normalized level of reserves is in excess of the optimal one. Hence, the study recommends reducing the international reserves to the optimal level. This is almost 25% of the current levels of reserves. We also present a scenario whereby the reserves are reduced by almost half of its existing levels in year 2007. We then invest a small portion, through a sovereign wealth fund, in VIX based options. In case of crisis (sudden stop) the payoff of the options will yield an amount that is almost the same as the kept value of reserves. Following this strategy allows for an opportunity gain in the range of (1-2) % of GDP. Moreover, the study proposes the activation of other institutions role in the society to attract long term investments, maintaining a certain limit of coordination among fiscal and monetary policies and consolidating the norms of ethical standards in the financial system.

"International reserves are both observable and have a market value..."
Gray, Merton and Bodie 2007

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1. Introduction

Recently, along with the dramatic increase in international reserves in emerging market countries, estimated by more than 60 percent since the Asian financial crisis in 1997 as mentioned by Mendoza (2004), debates about the fitting/optimal amount of reserves for an open economy have gained a new life (Jeanne and Rancière 2009). Being characterized by weak access to international capital markets, recurrent credit crunches and financial underdevelopment; emerging countries run persistent current account deficits and are vulnerable to foreign investors' insights about the underlying economic and institutional circumstances. Jeasakul (2005) put forward that, since investment decisions are based upon profitability criteria,[3] any alterations in these conditions due to financial crisis or turmoil, foreign creditors can suddenly cause capital inflows to come to an end, leading to what Dornbusch, Goldfajn and Valdes (1995) firstly marked as a "sudden stop", a term that was further developed analytically in Calvo (1998).

1.1 Sudden Stops: Some Stylized Facts

Many Asian countries during the late nineties of the last century were hit by currency crises and sudden stops. [4] Since then, hoarding international reserves has been the preferential policy of most developing economies (Cheung and Qian 2007; Wijnholds and Sndergaard 2007). Sighted in this light, the sharp augmenting in the amount of reserves held by many emerging markets can be relevant to the rise in the “globalization hazard” that confronts emerging markets as suggested by Calvo (2002). A moderate probability of globalization hazard “sudden stop” can induce emerging markets to self-insure fully by hoarding international reserves, rather than relying on non-reserve options of taking

[3] Martin and Morrison (2008) label these speculative flow of funds (or capital) from one country to another as being "Hot Money", because they can move very quickly in and out of market, hence potentially lead to market instability.

[4] Financial crises are events in which people suspect that some major economic agents can no longer uphold their financial obligations, which trigger panic in financial markets. Generally, the term financial crisis is used to describe currency crises, banking crises and debt crises. In the past, the scope of financial crises tended to be domestic. However, contagion effects became more prevalent in the past decades, as the international community witnessed a series of major financial breakdowns spanning various countries from the debt crises of 1982 to the Russian crisis of 1998 (Jeasakul 2005).

preventative measures. A sudden stop, [5] in this context, is a distinctive phenomenon of the crisis wracked in emerging countries during the post of the nineties. Lee (2008) put forward that sudden stops in its essence is a situation under which there is a reversal capital inflows to the country, lack of access to external insurance and hence a sharp current account adjustment on recipient countries. Caballero and Panageas (2005) demonstrate that international financial markets cause the sudden stop and not the emerging economies themselves.

On the main features of sudden stops, Mendoza (2008) mentioned three stylized facts that are: (1) Changing direction of international capital flows in a reverse order, (2) reductions in domestic production and absorption, (3) Alterations in asset prices. Besides these three main outstanding macroeconomic regularities, there are three distinct features of sudden stops: The first encompasses that they are placed within business cycles. Calvo, Izquierdo and Rudy (2005) points out that sudden stop describes a situation under which country's capital flows are insignificantly below their mean which is rarely events in each country. Secondly, they indicate business cycle asymmetries. Third, standard growth accounting displays that a large slump in the Solow residual accounts for a Sudden Stops’ initial output collapse. Part of this is due to factors that favor the Solow residual as a measure of “true” total factor productivity (TFP), such as changes in imported inputs, capacity utilization, and labor hoarding (Mendoza 2006).

In a nutshell, Asian crises revived the literature on sudden stops. Calvo (1998) in his work portrays the basic procedures and implications of sudden stops and Calvo and Reinheart (1999) document the incidence of the phenomenon. For emerging countries, having much of their growth ahead of them, any sudden stop in capital flows has serious repercussions on the economy and might very well trigger a recession. Caballero and Panageas (2005) pointed out that in a typical sudden stop, external funding declines by 10 percent or more, and the key impact lasts for over six years. The first considerable episode of this type was the Mexican crisis of 1994-1995, followed by the severe Asian crisis in 1997, where the hot money firstly moved out of Thailand and then from

[5] Jeanne and Rancière (2009) in their empirical study concerning assessing the optimal levels of international reserves for a group of emerging market countries identified a sudden stop at the point where the ratio of capital inflows to GDP falls by more than 5 percent.

the Southeast Asian countries. The Russian and the Brazilian crises in the late 1990s and beginning of the 21st century showed that this was not an exceptional event (Ito, 2006). Even well managed developed economies suffer from sudden stops or crashes. The recent 2008 financial crisis is a proof of this. The situation is even worse for emerging economies such as that of Egypt. Foreign direct investments (FDI) are needed to finance new long term projects especially in emerging markets where capital is hard to find. In the case of crash, at a moment notice these economies are required to face capital outflows. It is not an exaggeration to say that, in the mean time, sudden stops portray the profile of the world Cúrdia (2007).

1.2 International Reserves: Literature Review

International reserves, also popularly known as foreign exchange reserves, are those liquid assets that are nominated in different reserve units (foreign currency, foreign currency bonds and gold) and basically managed by monetary authorities; the central or reserve banks (Kester 2001; Bar-Ilan and Lederman 2006; IMF 1993). The issue of international reserves is not a new topic, but it has gained new eminence for two noticeable reasons: the dramatic surge in the level of such reserves over the last decade and the diversity of management styles. Alongside these developments; there is a rising debate about the extent to which hoarding excessive reserves is necessary. The opponents point out that holding a lot of reserves is costly; it yields a lower return than the interest rate on the country's long-term debt (Jeanne and Ranciere 2006; Aizenman and Marion 2002). Hence, there is no need to hold cash in the bank and pay high interest on outstanding liabilities.

On the other hand the proponents of holding many reserves claim that reserves play a key role in promoting the stability of any emerging configuration due to a sudden stop, counting up the benign impacts of accumulated reserves, even if it is not a panacea. *One rationale* is that reserves allow the country to self-insure against sudden stops, [6] capital

[6] Aizenman (2009) spot light on the fact that emerging markets that increased their financial integration during the 1990s-mid 2000s, hoarded international reserves due to precautionary motives, as self insurance against sudden stops, and deleveraging crises as suggested in the earlier research. Crisis-insurance motives could be considered as the main driver behind international reserve accumulation in emerging economies.

flight, mitigate real effective exchange rate effects of terms of trade shocks[7] and hedge against the risk of balance of payments crises, and therefore avoids being shut out of international capital markets, which came to be apparent as higher after the 1997-98 Southeast Asian crises (Jeanne and Ranciere 2006; Aizenman 2007[a, b]). Hence fore, having a precautionary stock for unexpected shocks promotes the nations capability to smooth domestic absorption in response to sudden stops, by allowing more persistent current account patterns. A competitive approach to that strand line of thought exhibits that international reserves accumulation is a by product of promoting export competitiveness, which results in better job creation particularly in the work of China as explained by Dooley, Folkerts-Landau and Garber (2004).

A *second view* postulates that reserves serve up as a deterrent against speculative attacks especially under a floating exchange rate regimes and deregulation of the foreign exchange rate markets; being equipped with reserve assets represents a safety valve of the nation against speculative attacks (Sidaoui 2005; Noyer 2007). In this context, accumulated reserves promote the countries ability to absorb any shock and hence its exchange rate vis-à-vis other currencies does not affect. A *third vision* states that countries based on hoarded reserves could manage international transactions imbalances directly and implement monetary as well as foreign exchange policies indirectly (Cheung and Qian 2007; Kester 2001; Noyer 2007; IMF 1993). A *fourth reason* mentioned the non-fundamental factors or psychological aspirations as motives for mounting up reserves. A fifth strand line focuses on the benefits of international reserves as a tool for sterilization intervention policy; where the central banks sells public bonds for international reserves during the capital flows boom to maintain internal stability (1.e. stability in prices) (Aizenman 2007[c]; Caballero 2000). Consequently, exchange rates will not be affected and the nation's exports will still be competitive. It is noticeable that accumulating growth to enhance

[7] The real effective exchange rate represents the weighted average of a country's currency relative to an index or basket of other major currencies adjusted for the effects of inflation. The weights are determined by comparing the relative trade balances, in terms of one country's currency, with each other country within the index. While the terms of trade denotes the relative prices of a country's export to import.

exports goes in consistency with the mercantilist school of trade (Aizenman and Lee 2005; Aizenman[b] 2007).

Other sights highlight the importance of reserves for countries to keep up with their peers and stimulate their growth given the positive impact of reserves on the output circumstances (Bar-llman and Lederman 2006; Kriesler and Cruz 2008). In this respect, the proponents of hoarding reserves claim out that reserve accumulation is not an end in itself but a mean to different ends; reserves give the country with large amount of reserves a competitive advantage over its peers. It follows up that assuming two economies with similar economic fundamentals, [8] in case of a financial crisis in the region, the one with a higher level of international reserves is less likely to be attacked and is more likely to survive the crisis. Hence for, the cost of building up excessive reserves is trivial relative to the economic consequences of a crisis.

1.3 International Reserves; is it the case of more is better or there is an opportunity cost?

Having discussed the importance of international reserves in the period of crisis, the question comes to mind: Is it a case of “more is better” or a case of “trade off”? Throughout this paper, we postulate that the international reserve accumulation process may achieve some relevant economic purposes up to a certain level, above which there will be an increasing opportunity cost. It is rather non-controversial to state that, ceteris paribas, international reserves help absorb unexpected exogenous triggered shocks and smooth current and capital account imbalances; in this regard its outstanding to mention that external crisis could be due to the response of international financial markets to national crisis (i.e. there may be national crisis that lead international markets to loose confidence in investing in emerging markets). The crisis experience and the development after it appear to be consistent with the notion of accumulating international reserves to anticipate future speculating. The

[8] Economic fundamentals refer to socio-economic variables that reflect the overall performance of the economy. For example; interest rates, exchange rates, the government’s budget deficit, the country’s balance of trade account (relating to exports and imports), the level of domestic business confidence, the inflation rate, the state of (and confidence in) the banking and wider financial sector and consumer confidence, health of job market, ability of wages to keep up with prices and value of home prices and stock prices (which have the greatest impact on household wealth).

debatable question, however, is regarding the optimal level of international reserves an economy has to hold?

Regarding the adequacy level of reserves, there are two schools: 1) Motivated by having low opportunities to penetrate international financial markets, Greenspan-Guidotti principle of maintaining reserves equivalent to existing short term obligations (short external debt maturing within one year) in case of quick drop in external financing circumstances (Jeanne and Rancière 2009; Guidotti 2002, IMF 2000; European Central Bank 2006). Such approach seems to be prominently followed by the Egyptian Central Bank since 1991 (Messrs, Opev and Ondiege 2000). An outstanding notice of the evolution of international reserves in Egypt is that it recorded an increasing upward trend that exceeds the short term obligations, see figure 1 in the annex. 2) Caballero and colleagues, on the other hand illustrate that there are other tools, the purchase of future options on the risky assets, which make it possible for emerging economies to penetrate international financial markets at a fraction of the held reserves. The payoff of the risky assets, at the time of sudden stop, would be equivalent to the needed reserves to offset the drop in the needed capital.

On reviewing the international experience in managing reserves, IMF (2008) puts forward that countries with large accumulated fiscal surpluses tend to establish two sovereign wealth funds (SWF). Such funds are mainly financed by foreign exchange revenues on commodity exports and/or transfers of foreign reserves from the central bank. These funds have raised concerns about: *(i)* Financial stability; *(ii)* corporate governance as well as enhancing transparency considerations versus commercial objectives; and *(iii)* finance expected future pension expenditure and sustain general government spending in case of an economic downturn political interference and protectionism or strategic and political objectives of such funds. Blundell-Wignall, Hu and Yermo (2008) defined SWFs as pools of assets owned and managed directly or indirectly by governments to achieve national objectives. SWFs[9] support developmental plans of the governments through: *(i)* Diversifying the investment portfolio; *(ii)* getting a better return on reserves; *(iii)* financing expected future pension expenditure *(v)* meeting future needs

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[9] Some of the longer-established SWFs, e.g., those of Kuwait, Abu Dhabi, and Singapore, have existed for decades (table 1 in the annex).

when natural resources run out; *(vi)* setting price stabilization schemes; *(vii)* promoting the industrialization process; and *(viii)* supporting strategic and political objectives and sustain general government spending in case of an economic downturn (Blundell-Wignall, Hu and Yermo 2008 and IMF 2008). It is worthwhile to mention that one of the investment funds retains a conservative strict investment policy and is controlled by a central bank, while the other is subject to more flexible rules under which the government is putting down a strategic asset allocation schema to boost the returns on the annual budget and hence be able to higher spending on priority requirements. We next present an overview of one of the commonly used indices to measure the volatility of the global financial market; the CBOE volatility index (VIX). Caballero and colleagues, and do are we; propose to invest in options that are related to the VIX.

1.4 Option Pricing Approach to International Reserves

Away from the conventional method of hedging against risk that encompasses full dependence on international reserves, this paper handles a new approach that is "option pricing".[10] The option pricing approach provides another tool that allows the government to decrease the amount of reserves to hedge against sudden stops in cash flows when a crisis occurs (Lee 2008). There are two pillars of option pricing approach: *(i)* To decrease the accumulated reserves to the optimal level; and *(ii)* to invest in risky assets that are VIX based, such investments which yields a payoff that compensates for the drop in the reserves.

The VIX is the ticker symbol for the Chicago Board Options Exchange (CBOE) Volatility Index, a popular measure of the implied equity market volatility extracted from options on S&P 500 firms (Caballero and Panageas 2005; Keel 2006). It is sometimes called the investor fear index, since investor uncertainty can lead to high market volatility through drops in prices. Options are traded on the VIX, enabling additional hedging and speculation positions on volatility. Closely

[10] The discussion of the equivalence between insurance and a put option was firstly appeared in Merton's (1976) analysis of the cost of providing deposit insurance. He noted that the Federal Deposit Insurance Corporation (FDIC) in the United States, provided guarantees for the loan extended by depositors to banks. Moreover, there was a further belief that the U.S. government offered the ultimate implicit guarantees for the liabilities of the FDIC, and thus those of the banks.

related to the VIX are the VXD, or CBOE Dow Jones Industrial Average Volatility Index, and the VXN, or CBOE NASDAQ 100 Volatility Index. VIX-based investments could yield a significant reduction in the average cost of sudden stops. This result should not be surprising to those following the practices of hedge funds and other leading investors. Except for extremely high frequency events, which unfortunately sudden stops are not, institutional investors seldom immobilize large amounts of “cash” to insure against jumps in volatility and risk-aversion. The use of derivatives and the creation of the VIX in particular, are designed precisely to satisfy hedging needs. Why should central banks, which aside from their monetary policy mandate are the quintessential public risk management institutions, not adopt best-risk-management practices?

1.5 The Objective of the Study

Caballero and Panageas (2004) derived and estimated a quantitative model to assess the (uncontingent) reserves management strategy typically followed by central banks. They concluded that this strategy is clearly inferior to one in which portfolios include assets that are correlated with sudden stops. In their model, they showed that holding contracts on the S&P100 implied volatility index provide emerging economies with a tool to well manage their reserves and make profit. In this study we adopt the methodology presented in Caballero and Panageas (2005) and apply it, after modifications, to the Egyptian economy. This paper, quantitatively, examines the optimal amount of reserves to be held by the Egyptian government so as to maximize the utility of domestic absorption (total expenditures on household consumption, plus government consumption plus investments), and calculates the corresponding optimal level of absorption. In this study we also estimate the optimal investments in risky assets. The payoff at the time of sudden stops will be enough to offset the reduction in the reserves. Thus the central bank will have enough reserves to smooth the drop in foreign capital flows. By developing optimal hedging strategies, Egypt could reduce its international reserve.[11]

[11] The paper does not pretend to know the objective function of certain central banks. Therefore, the notions of “adequacy” versus “excessive” presented in this paper are purely assessments of whether or not reserve accumulation exceeds the conservative estimates of reserves needed for self-insurance purposes.

This paper is organized as follows. In section II, we give a conceptual overview of the sudden stops and explain the basic features of the international reserves, upon which the quantitative model is based. In Section III, we show the optimal level of reserves that should be held by Egypt. The proposed model is used to analyze and forecast how the country could hedge against sudden stops. In Section IV, we present results, summary, and policy implications. There are three appendices that contain different elements of the mathematical techniques and the stochastic optimal control approach.

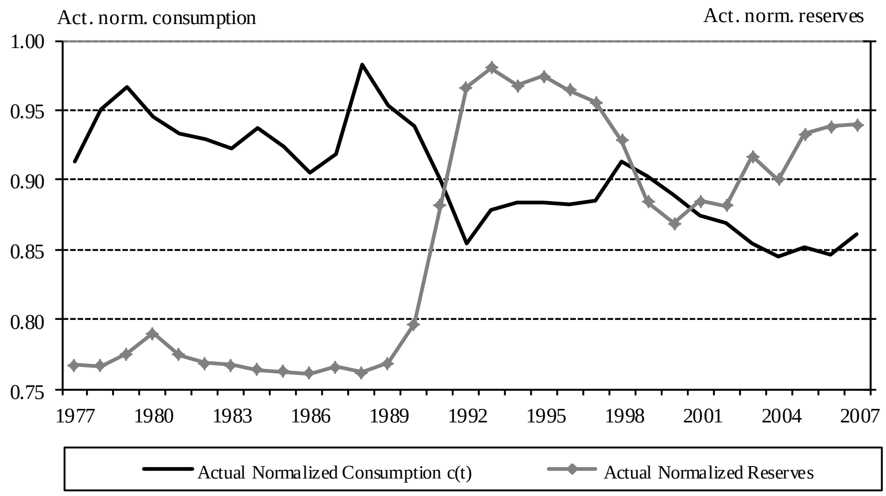
2. Sudden stops and the Mathematical Model

In this section, the current status of consumption and reserves for Egypt are put forward. We then present a dynamic equation for the demand for reserves that incorporates monetary disequilibrium considerations is derived following closely the approach presented in (Caballero and Panageas 2005, 2003) and using the method of optimal stochastic control to obtain our results. It is worthy to mention that this paper focuses on the crises that are triggered externally. Under the stringent assumption of time-invariance of the variables under study and the cost function, optimal levels of reserves and consumption are derived. The main bulk of the stochastic optimal control procedure stems from the work of Abutaleb and Papaioannou (2009).

2.1 Egypt Current Status

Figure 1 displays the fact that there is a negative relation ship between actual consumption and actual reserves. It is obvious that during the period (1977-1989) Egypt accumulated modest level of reserves as a percentage of GDP in relation to high level of consumption as a percentage of GDP. Years 1979 and 1980 were exception years where at the former the normalized reserves represent 0.02 and 0.97 for the consumption, but when reserves increase by 0.02 percentage points in year 1980, consumption decreases to 0.95 as a percent of GDP. Starting from year 1989 accumulated normalized reserves witnessed a dramatic increase at the expense of normalized consumption that fell behind; that situation which persisted till year 1992. Thereafter, normalized levels of both reserves and consumption were to some extent stable at higher and lower levels respectively till year 1997, after which normalized reserves started to get smaller for the favor of normalized current consumption till year 1999.

Figure.1 The Actual Normalized Consumption and the Actual Normalized Reserves



Source: Based on the authors calculations and data from WD1.

A remarkable feature of the profile of Egypt's normalized reserves as well as consumption is that throughout years 1999 and 2000 they moved together downwards, but normalized consumption are still at a higher level than normalized reserves. Over the period of (2001-2003) normalized levels of reserves and consumption seemed to move in divergence to each other, where the normalized level of reserves increased while that of consumption diminishes. Beginning from year 2004 till the end of the period of the study, international reserves increased gradually in contrast to normalized levels of consumption that looked like a zigzag pattern it its movement downward.

A sudden stop is defined as a drop in domestic absorption from the previous years. A sudden stop occurred at the 1981 (at the assassination of President Sadat) and continued for five years, the recovery occurred at 1986 till 1991; a sudden stop happened in 1991 and lasted till 1993. Another sudden stop occurred in 1998 till the year 2004. In 1991, a jump in the reserves as a percentage of GDP occurred and it seems that accumulating high level of reserves was policy of the central bank at that time.

2.2 The Utility Function

We study a representative agent economy with a responsible government that seeks to maximize the expected present value of the utility from consumption C(t):

$$E\left\{\int_{t_0}^{\infty} U\left(C(s)\right)e^{-r(s-t_0)}ds\right\} \qquad (2.1)$$

where r is the riskless interest rate and the discount factor, and t_0 is the initial time. The utility function U(C(t)) has many shapes, and in this analysis we shall use the popular constant relative risk aversion (CRRA) shape given as:

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma} \qquad (2.2)$$

2.3 Emerging Market Economies and World Capital Markets:

Let Y(t) represent the country's income in the pre-development phase, and assume that it follows the Geometric Brownian motion model. The stochastic differential equation (SDE) of Y(t) is given as:

$$dY(t) = \mu_Y Y(t)dt + \sigma_Y Y(t)dB(t) \qquad (2.3)$$

where B(t) is the Brownian motion or the Wiener process.

A country in its developing stage would like to borrow against its post development income. The potential financiers are: (1) World capital markets WCM, and (2) Specialists. Unlike the specialist, WCM have limited understanding of emerging markets hence they do not accept contracts that are related to the emerging economies. The country can also accumulate international assets X(t). Both assets and liabilities pay a return of *r* per unit of time.

2.4 Specialists and Sudden Stops

Specialists are investors who are familiar with the emerging markets at large and are willing to invest in many areas where the WCM refrain from investing in it. In practice they have equity investments, FDI, the riskiest tranches of GDP-indexed bonds, or toxic-waste more generally. Thus, during non-sudden stop times "NSS" or normal times, the maximum flow of resources received is " $\bar{f}Y(t)$ ":

$$\max f^{NSS}(t) = \bar{f} \tag{2.4}$$

During sudden stops, the maximum flow of resources received from the specialists is " $\underline{f}Y(t)$ " with:

$$\underline{f} < \bar{f} \tag{2.5}$$

Thus

$$f^{SS}(t) = \underline{f} \tag{2.6}$$

Define A(t) as the sum of income and contingent flows from specialists:

$$A(t) = \left(\theta^{NSS} 1\{NSS\} + \theta^{SS} 1\{SS\}\right) Y(t) \tag{2.7}$$

where $\theta^{NSS}(t) \le (1+\bar{f})$ (2.8)

$$\theta^{SS}(t) \le (1+\underline{f}) \tag{2.9}$$

$$1\{NSS\} = \begin{cases} 1 & \text{Country in normal times} \\ 0 & \text{elsewhere} \end{cases} \tag{2.10}$$

$$1\{SS\} = \begin{cases} 1 & \text{Country in sudden stop times} \\ 0 & \text{elsewhere} \end{cases} \tag{2.11}$$

Note that $\theta^{SS} < \theta^{NSS}$

The net assets accumulation or reserves X(t) is now described by:

$$dX(t) = [rX(t) - C(t) + A(t)]dt \tag{2.12}$$

The change in reserves is due to: (1) Interest on reserves at *r*%, (2) The difference between the "GDP + net foreign accumulation" and the consumption "A(t)- C(t)". The consumption, C(t), is defined as: the gross national expenditure or the domestic absorption, that is the sum of household final consumption expenditure, general government final consumption expenditure and gross capital formation.

2.5 The Calculation of the Optimal Level of Reserves

We shall study the case where the economy is normal i.e. no sudden stops (NSS) and find the optimal levels of reserves and the corresponding optimal levels of consumption.
Define

$$V\left(X(t_0), Y(t_0)\right) = \max_{C(s)} E\left\{ \int_{t_0}^{\tau^{SS}} e^{-r(s-t_0)} U(C(s))ds + \varphi\left(X(\tau^{SS}), Y(\tau^{SS})\right) \right\}$$

= value function in the normal state (2.13)

$\varphi\left(X(\tau^{SS}), Y(\tau^{SS})\right)$=Utility of the desired value of the Reserves at the onset of SS (2.14)

τ^{SS} = time to sudden stop which is random

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma} \tag{2.15}$$

Transition from normal times to sudden stops occurs with a constant hazard rate λ at the random time τ^{SS}. The constraints and the system dynamics are:

$$dY(t) = \mu_Y Y(t)dt + \sigma_Y Y(t)dB(t) \tag{2.3}$$

$$A(t) = \theta^{NSS} Y(t) \tag{2.11}$$

θ^{NSS} for developing countries is around 1.2 (Caballero and Panageas 2005)

$$dX(t) = \left[rX(t) - C(t) + A(t)\right]dt \tag{2.12}$$

$$X(t) \geq 0$$

In vector format, equations (2.3), (2.11) and (2.12) are written compactly as:

$$\begin{bmatrix} dX(t) \\ dY(t) \end{bmatrix} = \left\{ \begin{bmatrix} r & \theta^{NSS} \\ 0 & \mu_Y \end{bmatrix} \begin{bmatrix} X(t) \\ Y(t) \end{bmatrix} + \begin{bmatrix} -C(t) \\ 0 \end{bmatrix} \right\} dt + \begin{bmatrix} 0 \\ \sigma_Y Y(t) \end{bmatrix} dB(t) \qquad (2.17)$$

The country is faced with the decision of how much to consume $C(t)$ in order to maximize the utility function and at the same time ending with a desired level of reserves $X(\tau^{SS})$ with a utility function $\varphi\big(X(\tau^{SS}), Y(\tau^{SS})\big)$. The final value of reserves is defined by the decision maker. Reserves, $X(t)$, play the role of providing the country with resources during sudden stops. Accumulating reserves, however, is costly and deprives the economy from precious resources that could be used for development. The optimization problem is to find the optimal reserves and consumption paths and at the same time satisfying the level of the desired reserves at the end time τ^{SS}.

The Equations of Caballero and Panageas (2005)

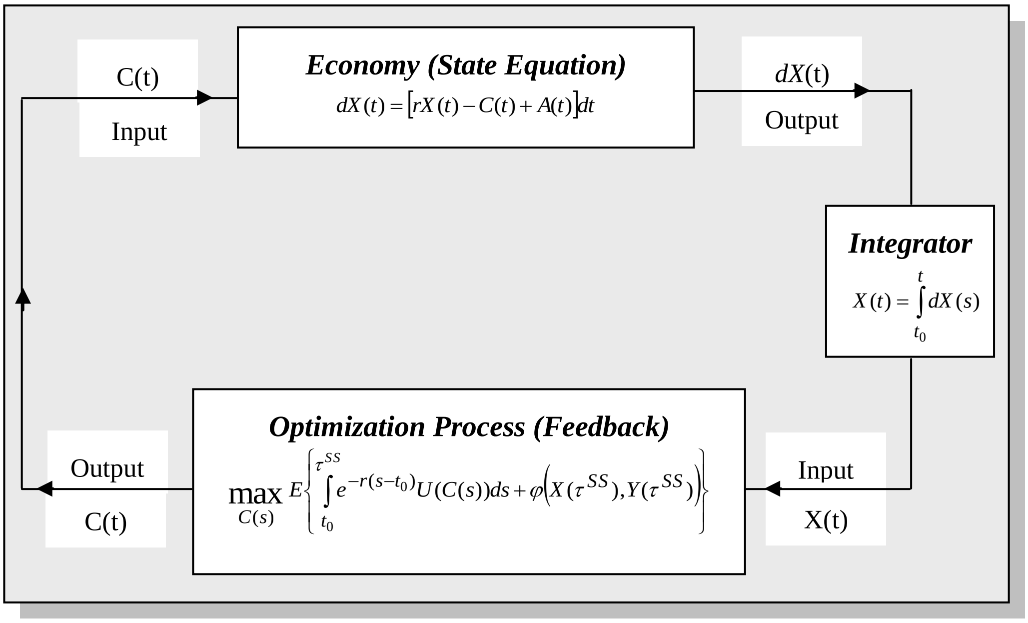
Assuming that time-invariance of the objective function, the optimal normalized consumption *c*(*x*) as function of the normalized *x*(t) reserves was derived as (see Appendix A):

$$c(x) = K \frac{x^{\frac{2}{\sigma_Y \gamma}}}{1 - x^{\frac{2}{\sigma_Y \gamma}}} \qquad (A.8)$$

where K is a constant to be determined from the boundary conditions. Notice that we see a positive correlation between the optimal normalized levels of consumption and the optimal normalized levels of reserves. Equation (A.8) is what is called the feedback equation.

Figure 2 maps out the relationship between consumption as well as international reserve levels. It highlights that, according to the economy SDE (equation 2. 12), there is a negative relationship between international reserves and consumption. Without doubt, there is a trade off; the more the money is allocated for reserves, the less is the expenditure on consumption and the vice versa. Considering that our goal is to maximize the expected value of the utility function, the resultant optimization equation (equation A. 8) reveals that there is a positive relation between the levels of reserves and the consumption (the feedback). Thus, a decrease/increase in reserves will be mirrored in drop/rise in consumption. This way we have two mechanisms acting on the economy; the first generates negative correlation between reserves and consumption and the second generated positive correlation between consumption and reserves. Thus, we have what is called a negative feedback mechanism and the economy is stable.

Figure.2 System Dynamics and Feedback Mechanism



3. Optimal Levels of Reserves In Case of Adding Risky Assets to the Portfolio

In this section, a quantitative framework is developed to bring forward the insurance motive of holding international reserves. The insurance value of reserves is quantified as the market price of an equivalent option that provides the same insurance coverage as the reserves. This quantitative framework is applied to calculating the cost of an international insurance arrangement (e.g., VIX) and to analyzing one leg of an optimal reserve-holding decision. It brings forward the approach of option pricing as a device that enhances Egypt capability to access international markets. Conceptually speaking, this approach goes in parallel with the insurance aspect of international reserves. Moreover, it steps further where the reserves are being recycled and hence widen the scope of available money to fund public expenditures on priorities in the future.

In this section we present the optimal levels of reserves, the corresponding optimal levels of consumption, and the optimal values of the optimal investments in risky assets. The derivations are shown, in detail, in Appendix B. We shall present the significant potential gains from improving current SS management practices by adding contingent instruments that are largely independent of the country's actions and sensitive to international events. One of the commonly used contingent instruments is based on the change in the volatility index VIX.

3.1 Portfolio Decision:

The international reserves of Egypt were in the order of 30% of GDP in 2007. We need to gradually reduce this amount to a fixed level on the span of few years. Purchasing risky assets that are VIX based is the key to that policy, such risky assets enjoy high values of payoff when SS occurs. At the same time, the payoff should be enough to ensure the coverage of several months of imports once an SS occurs. The basic idea is that starting with high value of reserves $X(t_0)$ we need to reduce this quantity gradually to a final fixed value X_C . At the time of SS there is a payoff obtained from an external fund X_F. The summation of X_C and X_F should be around 10 months of imports (the Greenspan doctrine). In this section we calculate the optimal reserves to be held by the central bank and the optimal values invested in risky assets, and at the same time, the utility of the consumption is maximized. The central bank reserves are gradually reduced to some fixed final value X_C in the case of a sudden stop. This policy will reduce the needed cashed reserves from the current high levels to the value X_C. The reduction in the reserves will free needed cash to be infused in the economy. This will act as a stimulus to the GDP growth. We first explain the mechanism of getting into SS.

Getting into SS

There is a Poisson process with intensity λ that puts the economy in a sudden stop zone at stochastic time τ^{SS} 1.e.

$$\mathrm{Prob}\{\text{SS occurs in the interval dt}\} = \lambda dt$$
$$\mathrm{Prob}\{\text{SS does not occur in the interval dt}\} = 1 - \lambda dt$$

where λ is the mean time of occurrences per unit time. Let N(t) define the independent Poisson process with the following properties:

$$\mathrm{Pr}\,obablity\{dN(t) = 1\} = \lambda dt$$
$$\mathrm{Pr}\,obablity\{dN(t) = 0\} = (1 - \lambda)dt$$

Thus, $E\{dN(t)\} = \lambda dt$

Assume that there is a financial asset with payoff F(t) that has the potential to exhibit a jump, F(t)J, in its value when the system enters the "SS zone" at the random time τ^{SS}. J is a nonnegative random variable that could have the probability density function of the lognormal distribution or others (Law and Kelton 1991).

The risky asset's payoff process is defined as:

$$dF(t) = r_F F(t)dt + F(t)JdN(t) \qquad (3.1)$$

The asset has a return $\dfrac{dF(t)}{F(t)}$.

In what follows we condition on those times τ^{SS} where the country enters into "SS zone". The addition of a risky asset with the above properties modifies the analysis given in Section 2. The evolution of the reserves now becomes:

$$dX(t) = \left[r\left(X(t) - \xi(t)F(t)\right) - C(t) + \theta^{NSS}Y(t)\right]dt + \xi(t)dF(t) \qquad (3.2)$$

Where $\xi(t)$ is the amount (or more precisely the number of units) invested in the risky asset F(t). Substitute the expression for the risky asset "$dF(t) = r_F F(t)dt + F(t)JdN(t)$" in the reserves equation we get:

$$dX(t) = \left[r\left(X(t) - \xi(t)F(t)\right) - C(t) + \theta^{NSS}Y(t)\right]dt + \xi(t)\left\{r_F F(t)dt + F(t)JdN(t)\right\}$$

Rearrange we get:

$$dX(t) = \left[rX(t) - C(t) + \theta^{NSS}Y(t) + \left(r - r_F\right)\xi(t)F(t)\right]dt + \xi(t)F(t)JdN(t) \quad (3.3a)$$

In a vector form, the new SDE describing the economy will be:

$$\begin{bmatrix} dX(t) \\ dY(t) \\ dF(t) \end{bmatrix} = \left\{ \begin{bmatrix} r & \theta^{NSS} & (r - r_F)\xi(t) \\ 0 & \mu_Y & 0 \\ 0 & 0 & r_F \end{bmatrix} \begin{bmatrix} X(t) \\ Y(t) \\ F(t) \end{bmatrix} + \begin{bmatrix} -C(t) \\ 0 \\ 0 \end{bmatrix} \right\} dt + \begin{bmatrix} 0 \\ \sigma_Y Y(t) \\ 0 \end{bmatrix} dB(t) + \begin{bmatrix} \xi(t)F(t)J \\ 0 \\ F(t)J \end{bmatrix} dN(t) \quad (3.4)$$

The objective function is now defined as:

$$V\left(X(t_0), Y(t_0), F(t_0)\right) = \max_{C(s), \xi(s)} E\left\{ \int_{t_0}^{\tau^{SS}} e^{-r(s-t_0)} U(C(s))ds + \varphi_1\left(X(\tau^{SS})\right) + \varphi_2\left(\xi(\tau^{SS})F(\tau^{SS})\right) \right\} \quad (3.5)$$

Where $\varphi_1\left(X(\tau^{SS})\right) = \left[X(\tau^{SS}) - (X_C + X_F)\right]^2$

$$\varphi_2\left(\xi(\tau^{SS})F(\tau^{SS})\right) = \left[\xi(\tau^{SS})F(\tau^{SS}) - X_F\right]^2$$

X_C is the desired value of the risk free assets or cash for the reserves
And X_F is the desired payoff from the risky asset

The objective is to find the control variables $C(s), \xi(s)$ that will maximize the expected value of the utility function and at the same time make the expected values of the reserves, from risk-free assets at the terminal time, as close as possible to X_C and the expected value of the payoff of the risky assets, at the terminal time, as close as possible to X_F. As it stands, this is not an easy problem to solve. An analytic solution is almost impossible and one has to go to numerical solutions or use some approximations to make the problem analytically tractable.

3.2 Normalized Variables and the Optimal Solution

As we prove in Appendix C, the normalized reserves $x(t) = \dfrac{X(t)}{\theta^{NSS} Y(t)}$ have the SDE:

$$dx(t) = \left[\left(r - \mu_Y + \sigma_Y^2\right)x(t) + 1 + (r - r_F)\bar{\xi}(t) - c(t)\right]dt - \sigma_Y x(t) dB(t) + \bar{\xi}(t) E_J\{J\} dN(t) \quad \text{(C.7)}$$

where $c(t) = \dfrac{C(t)}{\theta^{NSS} Y(t)}$ is the normalized consumption, and $\bar{\xi}(t) = \dfrac{\xi(t) F(t)}{\theta^{NSS} Y(t)}$ is the normalized payoff due to investments in risky assets. As shown in Appendix B, the approximate optimal values for the normalized consumption, c(s), and the normalized value of risky assets are given by:

$$c(s) \approx \left(\alpha K_g\right)^{\left(\frac{-1}{\gamma}\right)} \exp\left[\left(\frac{-1}{\gamma}\right)\left(r(s-t) + \lambda s + \alpha x(s)\right)\right] \quad \text{(B.26)}$$

$$\bar{\xi}(s) = \frac{\xi(s) F(s)}{\theta^{NSS} Y(s)} \approx \frac{1}{\alpha E_J\{J\}} \left\{ \ln\left[\frac{(r_F - r)}{\alpha E_J\{J\} \lambda K_g}\right] - \left[\lambda s + \alpha x(s)\right] \right\} \quad \text{(B.27)}$$

The above are approximate formulae. They give us the approximate structure or shape of the optimal functions. Thus, in general, one can assume that:

$$c(t) = K_c \exp\left[\left(\frac{-1}{\gamma}\right)\left(\lambda_c t + \alpha_c x(t)\right)\right] \quad \text{(3.6)}$$

and

$$\bar{\xi}(t) = \frac{\xi(t) F(t)}{\theta^{NSS} Y(t)} = K_{\xi 1}\left\{K_{\xi 2} - \left[\lambda_c t + \alpha_c x(t)\right]\right\} \quad \text{(3.7)}$$

where K_c, λ_c, α_c, $K_{\xi 1}$ and $K_{\xi 2}$ are unknown constants to be determined by maximizing the utility function $U(c) = \dfrac{c^{1-\gamma}}{1-\gamma}$, when γ is large 1.e. $\gamma >> 1$.

Comments

As we saw in section II, it is interesting to notice that there is a negative feedback mechanism that stabilizes the economy at optimal levels of reserves and consumption. Specifically, the normalized reserves x(t) will decrease as the normalized consumption c(t) increases (see equation (C.7)) . Because there is another relation, stemming from the optimization or feedback (eqn. (3.6)), the consumption will decrease as the reserves decrease (K_c is positive and α_c is negative). When this happens, the reserves will increase (eqn. (C.7)), and consequently the consumption will increase (eqn. (3.6)). Thus, we reach a dynamic equilibrium and the system (economy) is stable.

3.3 The Time of Sudden Stop τ^{SS}

For an accurate analysis, the sudden stop time τ^{SS} is a random quantity. According to the analysis of SS in the past years, it was found that an SS occurs almost every 4-6 years (Caballero and Panageas 2005). It is costly if the SS occurs and we are not prepared by buying risky assets. Thus, in this report we adopt the conservative policy of assuming that τ^{SS} = 4 years, and the government of Egypt must purchase the risky assets before the four years from the last SS. Thus, we shall present the scenario when the τ^{SS} = 4 years.

4. Results for the Egyptian Economy

This section is devoted to present the results of the study concerning the optimal level of reserves, consumption and the optimal value of investing in risky assets. The basic assumption of our analysis is that actual reserves are reduced to 10 percent of the GDP at the onset of SS. We shall first explain how to calibrate the model. Then, we display the optimal level of normalized consumption and the normalized reserves versus their actual levels. Next, under the assumption that there is a sudden stop during the next four years, we present the future values of reserves, risky assets investments, and the corresponding normalized consumption. Finally, we estimate the opportunity gain resulting from infusing the freed reserves, after excluding a small portion for investing in VIX based risky assets, in the economy to stimulate it.

4.1 Model Calibrations

The obtained approximate formulae of the normalized optimal reserves, the normalized optimal consumption, and the normalized optimal values of the risky assets will serve as a starting point to model the different variables. It has been observed that the Egyptian economy has two different periods. The first from 1977-1990, and the second period is 1991-2007. Thus, we use the same formulae but with different constants in each period.

Thus, we have for the first period, we have the equations:

$$c(t) = K\_c_1 \exp\left[\left(\frac{-1}{\gamma}\right)\left(\lambda_{c1} t + \alpha_{c1} x(t)\right)\right] \tag{4.1}$$

where K_c_1, λ_{c1} and α_{c1} are unknown constants to be determined by maximizing the utility function $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$, and γ was set to the value 9 >> 1.

in the second period we have a similar equation:

$$c(t) = K\_c_2 \exp\left[\left(\frac{-1}{\gamma}\right)\left(\lambda_{c2} t + \alpha_{c2} x(t)\right)\right] \tag{4.2}$$

where K_c_2, λ_{c2} and α_{c2} are unknown constants to be determined by maximizing the utility function $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$.

for the future values of c(s), we have:

$$c(t) = K\_c_F \exp\left[\left(\frac{-1}{\gamma}\right)\left(\lambda_F t + \alpha_F x(t)\right)\right] \tag{4.3}$$

where K_c_F, λ_F and α_F are unknown constants to be determined by maximizing the utility function $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$.

For each time period, the starting values for the reserves were different. To find the optimal reserves and the optimal consumption, we used the true values of reserves as initial conditions. Thus, for the first period we used *x*(1977) as initial conditions, for the second period we used *x*(1991) as initial conditions, and for the future period we used *x*(2008) as initial conditions. The future values of the investments in the risky assets,

$\bar{\xi}(t) = \dfrac{\xi(t)F(t)}{\theta^{NSS}Y(t)}$, were calibrated in the same way. We used the derived

approximate formula as a guideline. Thus,

$$\bar{\xi}(t) = \frac{\xi(t)F(t)}{\theta^{NSS}Y(t)} = K_{\xi 1}\left\{K_{\xi 2} - \left[\lambda_F t + \alpha_F x(t)\right]\right\} \tag{4.4}$$

where $K_{\xi 1}$, $K_{\xi 2}$, λ_F, and α_F are constants to be estimated by maximizing the utility function.

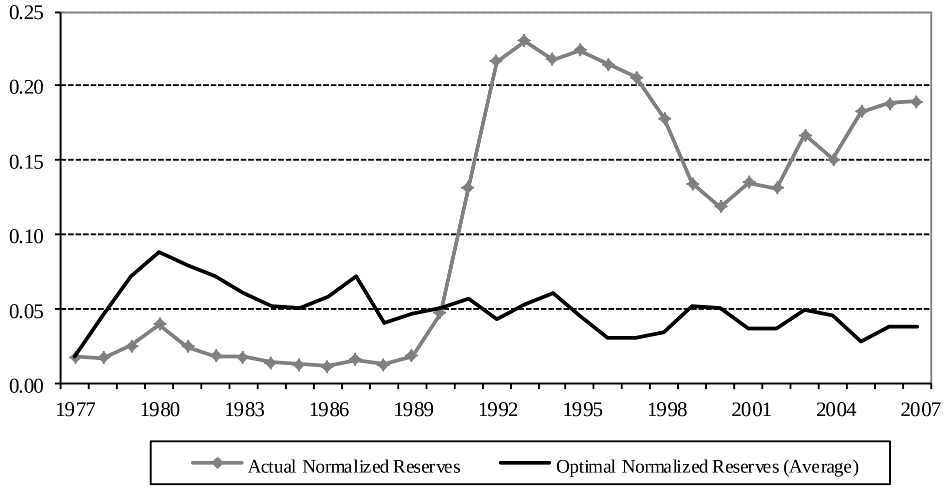
In all the periods, we used the same reserves SDE that was derived before:

$$dx(t) = \left[\left(r - \mu_Y + \sigma_Y^2\right)x(t) + 1 + (r - r_F)\bar{\xi}(t) - c(t)\right]dt - \sigma_Y x(t)dB(t) + \bar{\xi}(t)E_J\{J\}dN(t) \quad \text{C.7)}$$

During simulation, however, dN(t) was set to zero except at the time of occurrence of the sudden stop. The SS was simulated to occur in the year 2012. It was also assumed that $E_J(J) = 10$ 1.e. the average value of the random amplitude of the jump process is 10. Other values could also be used. In what to follow we present the average of ten simulation runs.

4.2 Actual Normalized versus Optimal Normalized Levels of Reserves and Consumption

Figure.3 The Actual Normalized Reserves and the Optimal Normalized Reserves



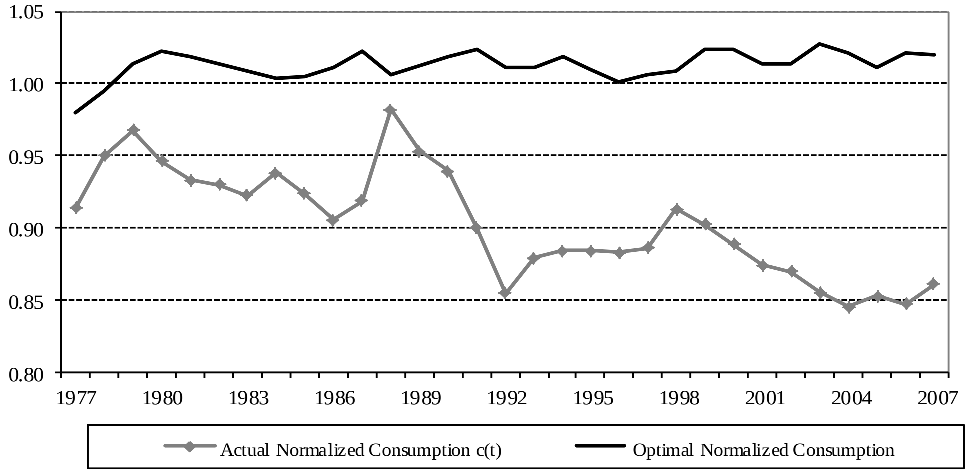
Source: Based on the authors calculations and data from WD1.

Examining the evolution of actual and optimal level of reserves in the Egyptian economy during the period 1977-2007, it is clear from figure 3 that there is a gap among both levels. Starting from 1977 till year 1989, the optimal level of reserves exceeds the actual ones. After that, the picture is reversed; where the actual reserves were much higher than the optimal values. For efficient management of international reserves, we recommend for the government of Egypt to decrease its accumulated reserves to the optimal level. The average optimal normalized reserves value is around 5% which is consistent with the observed values of reserves in developing countries. As a matter of fact; actual reserves have to down to almost a quarter of its existing rates in year 2007 as portrayed. The decision of diminishing the accumulated level of reserves encompasses two issues:

a- Is there an alternative policy that compensates the central bank for the drop in reserves and hence allows it to self-hedge against sudden stops in capital flows? If yes, what is this; and b- Is there an opportunity gain from reducing the actual normalized level of reserves to the optimal level?

The answer is yes for both questions as will be tackled in more details in the following two sub-sections respectively.

Figure.4 The Actual Normalized Consumption and the Optimal Normalized Consumption



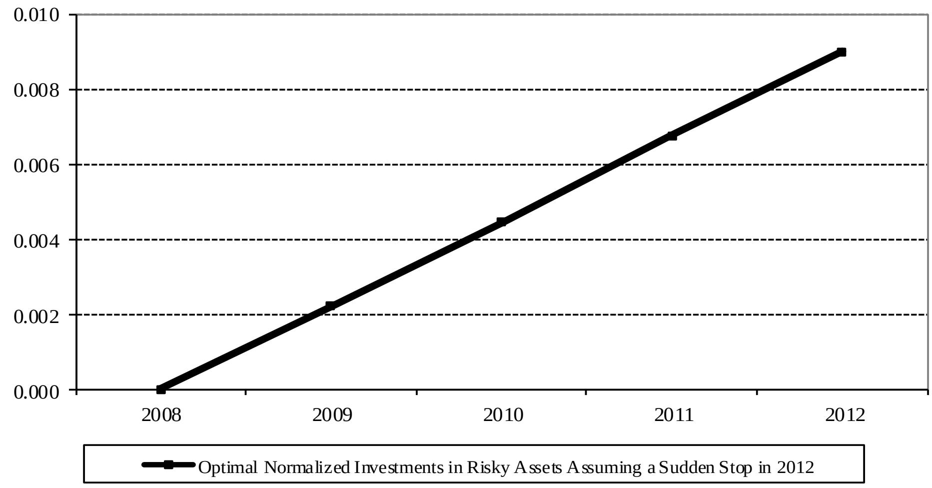
Source: Based on the authors calculations and data from WD1.

Figure 4 shows that, on the average, if we have followed the optimal path in managing the international reserves, the GDP would have been from 10 to 15 percent higher than the actual values. That is managing international reserves away from the optimal criteria; the actual normalized consumption is below its optimal levels all over the period of the study. This is under the assumption that the current account is the same during the whole period of the study. It follows that the Egyptian economy incurs an opportunity cost represented in enjoying low levels of welfare by its individuals, due to pooling large amounts of reserves.

4.3 Investing in VIX Based Risky Assets that are Contingent to Option Pricing Approach

Investing in risky assets through VIX based options is the complementary policy of reducing the actual level of reserves to the optimal one. Purchasing of risky assets that are VIX based allows Egypt to compensate for the drop in international reserves and obtain the needed funds when the sudden stop occurs. Making use of the approach of investing in risky assets that are VIX based, figure 5 gives an idea about the optimal normalized investments in risky assets such that utility of consumption will be maximized. Supposing that there is a sudden stop in year 2012, figure 5 indicates the optimal level of investments in risky assets needed to yield an equivalent amount to that dropped in the reserves to hedge against risks. It is worth to mention that the optimization criteria in our case is to find the optimal investments such that the welfare of the society will be maximized. In other applications as in portfolio management, the optimization criteria is to find the optimal investments such that return on investments will be maximized like in capital asset pricing model (CAPM) and arbitrage princing model (APA) (Standard and Poor's 2008).

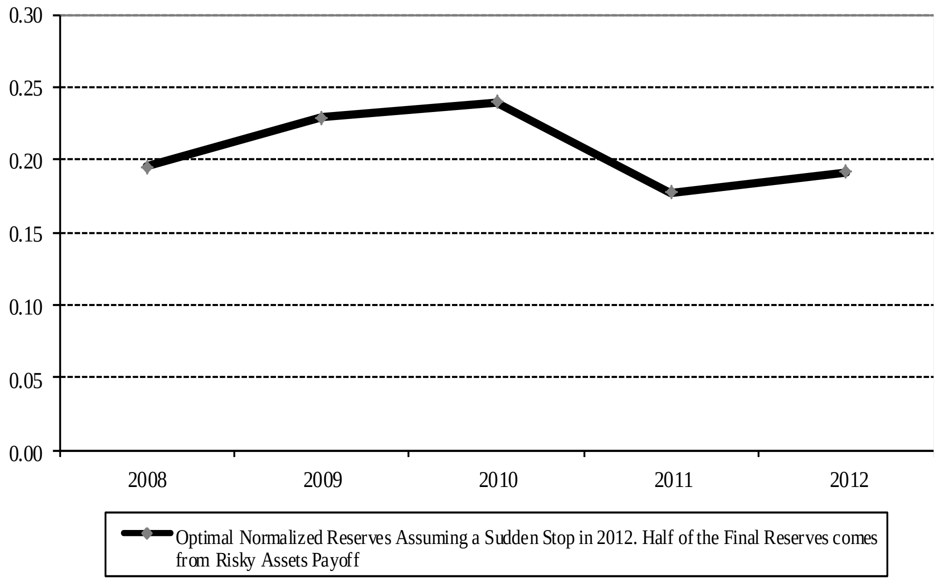
Figure.5 The Optimal Normalized Investments in Risky Assets Assuming that a Sudden Stop will Occur in Year 2012



Source: Based on the authors calculations and data from WD1.

Figure 6 marks out the optimal normalized reserves that could have been accumulated in 2010. It is worthy to mention that half of the final reserves comes from VIX based risky assets payoff, taking in to consideration the exclussion of 50 percent of it for being invested in VIX based risky assets.

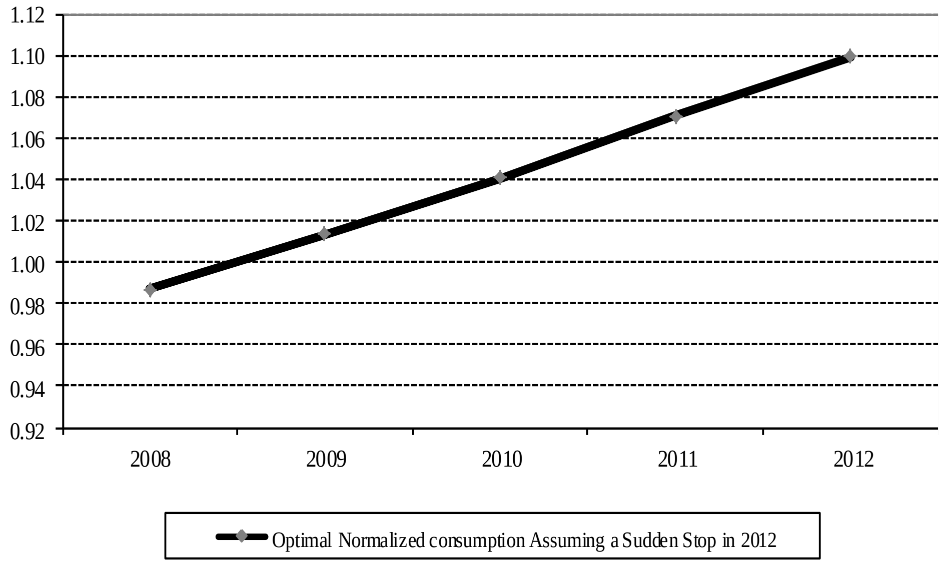
Figure.6 The Optimal Normalized Reserves Assuming a Sudden Stop Occurs in Year 2012 (Half of the Reserves was Constrained to Come from the Payoff, at 2012, on the Investments in the Risky Assets)



Source: Based on the authors calculations and data from WD1.

In consistent with the previous analysis, figure 7 predicts the optimal normalized consumption during the period 2008 till 2012 where the SS is expected to occur. Managing international reserves according to the optimal levels and applying the approach of option pricing, figure 7 sketches out that the normalized consumption will reach its optimal in year 2012 where the SS occurs.

Figure7. Optimal Normalized Consumption Assuming a Sudden Stop Occurs in Year 2012



Source: Based on the authors calculations and data from WD1.

4.4 Estimating the Opportunity Gain from Reducing Actual International Reserves

Under the assumption that there is a negative relationship between reserves and consumption; the Egyptian economy is assumed to gain a slight favorable increase in its GDP growth as a result of cutting down its actual reserves to the optimal level. It follows that reducing the normal level of international reserves to the optimal one is not the end of the story, since this released money could be injected in the economy to stimulate its growth. This subsection examines and quantifies the contribution of the released reserves to the GDP growth of the economy. Arguing that actual level of reserves should decrease to its optimal levels (to almost a quarter of its existing levels in 2007), we quantify the impact of infusing such released money (estimated to be 24.08 billion $) into the Egyptian economy according to a number of scenarios:

4.4a- (Scenario 1): Estimating the opportunity gain from infusing the total amount of the released reserves in the Economy

Table 1. Estimating the opportunity gain resulting from infusing all the released reserves in the Economy

Year	Y_i	M_0	ΔY_i	Y_{total}	$\%\Delta Y_{total}$	ΔGDP_{DGR} (% points)
2007	132.95	4.01				
2008	139.09	4.01	4.01	143.11		
2009	145.53	4.01	8.21	153.74	7.43	2.80
2010	152.26	4.01	12.60	164.86	7.24	2.61
2011	159.30	4.01	17.20	176.50	7.06	2.43
2012	166.67	4.01	22.23	188.90	7.02	2.40

Source: Based on authors calculations

Notes

$\boldsymbol{Y}_i$: Initial GDP in billions US$, GDP growth rate is assumed to be constant and it is equal to 0.046 at all years.

$\boldsymbol{M_0}$: Yearly equal share of released international reserves in billions US$.

$\Delta \boldsymbol{Y_i}$: Change in the initial GDP as a result of injecting all the released reserves in the economy.

$\boldsymbol{Y_{total}}$: Total GDP is the sum of initial GDP *(Y_i)*, and change in it *(ΔY_i)*; taking into consideration the effect of infusing all released reserves in the economy.

$\%\Delta \boldsymbol{Y_{total}}$: Percentage growth rate of total GDP *(Y_{total})*.

$\Delta \boldsymbol{GDP_{GR}}$ ***(percentage points)***: Measures the difference between the initial GDP growth rate which is equal to a fixed rate that is 4.62% and that GDP growth rate resulting from the injection of the total released reserves into the economy (ΔY_{total}).

Table 1 shows that a gradual infusion of the total released reserves in to the economy will contribute to GDP growth rate by a range from 7.02 to

7.43%. It follows that the new growth rate of GDP exceeds the old level by a range from 2.40 to 2.80 percentage points.

4.4b- (Scenario 2): Estimating the opportunity gain from investing one billion $ in VIX based risky assets and infusing the other in the Economy

Table 2. Estimating the opportunity gain resulting from investing one billion$ in VIX based risky assets and infuse the rest in the economy

Year	Y_i	M_0	ΔY_i	Y_{total}	$\%\Delta Y_{total}$	ΔGDP_{DGR} (% points)
2007	132.95	132.95	3.85			
2008	139.09	139.09	3.85	3.85		
2009	145.53	145.53	3.85	7.87	7.32	7.32
2010	152.26	152.26	3.85	12.08	7.13	7.13
2011	159.30	159.30	3.85	16.49	6.97	6.97
2012	166.67	166.67	3.85	21.31	6.93	6.93

Source: Based on authors calculations

Notes

$\boldsymbol{Y}_i$: Initial GDP in billions US$, GDP growth rate is assumed to be constant and it is equal to 0.046 at all years.

$\boldsymbol{M_0}$: Yearly equal share of released international reserves in billions US$.

$\Delta \boldsymbol{Y_i}$: Change in the initial GDP as a result of injecting 23.08 billions$ of the released reserves in the economy.

$\boldsymbol{Y_{total}}$: Total GDP is the sum of initial GDP *(Y_i)*, and change in it *(ΔY_i)*; taking into consideration the effect of infusing 23.08 billions$ of the released reserves in the economy.

$\boldsymbol{\%\Delta Y_{total}}$: Percentage growth rate of total GDP *(Y_{total})*.

$\Delta \boldsymbol{GDP_{GR}}$ ***(percentage points)***: Measures the difference between the initial GDP growth rate which is equal to a fixed rate that is 4.62% and that GDP growth rate resulting from injecting 23.08 billions$ of the released reserves in the economy (ΔY_{total}).

Table 3, by the same token, shows the impact of gradually infusion of 23.08 billions US$ of the total released reserves in the economy. The GDP growth rate is assumed to increase by a range from 7.02 to 7.43%. Such increase yields an increase estimated to be in the range of 6.93 to 7.32 percentage points above its old levels.

4.4c- (Scenario 3): Estimating the opportunity gain from infusing half of the total of the released reserves in the Economy, and invest the other half in risky assets that are VIX based

Table 3. Estimating the opportunity gain resulting from infusing half of the total released reserves in the Economy, and investing the other half in risky assets that are VIX based

Year	Y_i	M_0	ΔY_i	Y_{total}	$\%\Delta Y_{total}$	ΔGDP_{DGR} ((% points)
2007	132.95	2.01				
2008	139.09	2.01	2.01	141.10		
2009	145.53	2.01	4.11	149.63	6.05	1.42
2010	152.26	2.01	6.30	158.56	5.97	1.34
2011	159.30	2.01	8.60	167.90	5.89	1.27
2012	166.67	2.01	11.12	177.78	5.89	1.26

Source: Based on authors calculations

Notes

$\boldsymbol{Y}_i$: Initial GDP in billions US$, GDP growth rate is assumed to be constant and it is equal to 0.046 at all years.
$\boldsymbol{M_0}$: Yearly equal share of released international reserves in billions US$.

$\Delta \boldsymbol{Y_i}$: Change in the initial GDP as a result of injecting half of the released reserves in the economy.

$\boldsymbol{Y_{total}}$: Total GDP is the sum of initial GDP *(Y_i)*, and change in it *(ΔY_i)*; taking into consideration the effect of infusing half of the released reserves in the economy.

$\boldsymbol{\%\Delta Y_{total}}$: Percentage growth rate of total GDP *(Y_{total})*.

$\boldsymbol{\Delta GDP_{GR}}$ ***(percentage points)***: Measures the difference between the initial GDP growth rate which is equal to a fixed rate that is 4.62% and that GDP growth rate resulting from injecting half of the released reserves in the economy (ΔY_{total}).

Table 3 illustrates that investing half of the released in VIX based risky assets and gradually infuse the other half in the economy will stimulate the growth rate of the economy by a range from 5.89 to 6.05%. Such increase in GDP is assumed to be above its old levels by a range from 1.26 to 1.42%. We adopt such scenario in demonstrating the approach of VIX based risky assets; see figures 6 and 7.

5. Conclusion and Discussion

In response to the rebirth of the old debate concerning the appropriate level of reserves for an emerging open economy, this paper derives a simple formula for the optimal level of international reserves in Egypt. Surveying the literature marked out the various benevolent benefits of accumulating reserves especially in case of unanticipated sudden stops of capital flows. The question comes to mind is: Does holding up reserves is a case of “more is better” or there is a “trade off”? Unlike to the prevalent approach concerning the optimal management of international reserves in emerging countries that are based upon Greenspan-Guidotti-IMF principle, this paper adopts a new approach that encompasses the purchase of option contracts on VIX based risky assets. VIX-based investments could yield a significant reduction in the average cost of sudden stops. Options are traded on the VIX, enabling additional hedging and speculation positions on volatility. Throughout this paper, we postulate that the process of building up international reserves is necessary but not sufficient condition to ensure improvement in the standards of living of the society and maximizing its welfare. The main idea is how far/close the actual normalized level of reserves to the optimal normalized one is. The closer there are; the more is the utility of consumption of the society. But as long as there is a deviation among the optimal and actual normalized levels; whether it is above or below it, there is an opportunity cost that the society incurred. Enjoying excessive levels of reserves, we recommend for Egypt the following:

i) To scale down its pooled actual reserves to the optimal level; (subsection 4.2).

ii) To invest in VIX based risky assets which in turn compensates it for the reduction in reserves. Such investment yields a payoff sufficient to overcome disruptions resulting from sudden stops in capital flows. Consequently, it could smooth its consumption and investment expenditures (absorption); (subsection 4.3).

On the ground of those recommendations, the following policy implications could be derived:

5.1 Improving the welfare of the society

Under the assumption that there is a negative relationship between the total amount of reserves and the welfare level of individuals in the society, reducing actual level of reserves will improve the welfare of the society. Since the released reserves could be used for stimulating the economy; (see section 4.4).

5.2 Establishing investment funds or corporations outside the central bank to manage the released reserves

Following the VIX based options approach encompasses the establishment of an investment fund/corporation to manage the released reserves outside the central bank. On reviewing the international practices of establishing such investment funds/corporations, we found the following remarkable features (IMF 2008):

a- SWFs are operated outside the central bank: Despite the prominent experience of the public sector investment managers (such as reserve managers) in fixed income markets, most SWFs employ external expertise. The main driving force behind that is the limited capacity of the national public sector reserve managers for investment in other asset categories (IMF 2008). In this context; SWFs are operated outside the central bank, based on surveying international practices.

b- SWFs invest in a broad variety of assets; motivated by individual objectives: For countries with reserves pools in surplus of immediate central bank short term liquidity needs like the case of Egypt, creating a separate SWF can allow some released money to be managed with higher returns to compensate part of the cost of maintaining large liquidity stocks. While other SWFs are provoked by income purposes, and may invest across all major asset classes. Hence, the procedures of such funds have to be monitored to ensure that they are consistent directly or indirectly with the government's developmental plan.

c- There is no need to establish a unique investment fund: An outstanding feature of investment fund/corporations is that some countries like Kuwait, Singapore and Russia have two investment funds. Taking in to concern the political, economic and social considerations in the Egyptian context, this paper states that SWFs should be subject to the control of regulatory institutions that should be streamlined and consolidated. The question now is to what extent the Egyptian financial system could invest in risky assets? Despite significant efforts to upgrade the financial system, there is lack in technically proficient human resources. In this regard we recommend for Egypt to begin externally outsource a renowned international investment corporation that undertakes the management of freed reserves, till it acquires highly effective national human resources base.

5.3 Establishing of an Early Warning and Monitoring System at the International Level

Adopting the view of reducing international reserves to almost half and invest the remaining part in risky assets is no doubtfully a risky decision for a conservative country like Egypt. Building an early warning and monitoring system EWAMS to detect crisis at the international level becomes vital to avoid the crisis.[12] An early warning system can be defined as a system of data collection to monitor the probability of crisis onset (USAID 2007). EWAMS is useful in alarming the crisis in its initial phase and hence the necessary precautions and remedies could be taken. However, early warning of crisis is no guarantee of the prevention

[12] The focus on crisis at the international crisis does not mean that those at the intra-national level are insignificant. Rather, there is a critical need for having an integrated warning system that could predict crisis of all levels (internal, external, military, environmental, food…).

of crisis, since capacity and willingness to respond is essentially (Lundin 2004; USAID 2007; Campbell and Meier 2007). It follows that early warning and monitoring system will only function if the whole chain of response is in place (1.e. there must be an effective response to early warning signs).

5.4 Coordination between monetary and fiscal policies

The degree of coordination among various institutions in the society is an issue of a paramount interest from different categories in the society. The central bank being the only formal institution responsible for implementing the monetary policy does not work in isolation from other institutions in the society. Instead its decisions affect the policies of other institutions and vice versa. In this respect, to avoid the problems that may be resulting from making the decision of reducing international reserves. Other institutions in the society should do its best; each according to its responsibility. For example, the ministry of investments and its followed associations should do their best in maintaining the same level of investments especially in case of crises, which could be motivated by giving incentives. In a word, the formulation of consistent and rigorous based economic policies is a sort of an art where each decision has its resonance in other policies.

5.5 Culture, Ethics and Finance in the Financial System

Assessing an ethical culture among organizations is a key ingredient to business life in the organization (Argandoña 1998). Correcting unethical behavior once occurred is vital for healthy financial system. So, transparency, monitoring and accountability (1.e. the existence of corporate governance with its three pillars) should be in place. On the other hand, government policies, rather than stifling innovation and trading, should enjoy a reasonable level of transparency which in turn reduce the costs of the financial system (American Chamber of Commerce in Egypt, 2004). Furthermore, the degree of derivatives markets' upgrading needs to be considered. It follows that there are prerequisites that must be available in the workplace of the financial system to be well managed.

5.6 Future Work

This paper postulates that investing in risky assets, that are VIX based, is a beneficial tool that allows the government to efficiently manage its international reserves. More work is needed to identify which risky assets and how much to be invested. This might be the subject of another study.

APPENDICES

Appendix A: Optimal Consumption

In this appendix we derive the consumption C(t) that will optimize the expected value of the utility function subject to the system dynamics constraints. Instead of working with the reserves X(t), we shall, for mathematical purposes, use the normalized reserves, x(t), and the normalized consumption, c(t), defined as:

$$x(t) = \frac{X(t)}{\left(\theta^{NSS} Y(t)\right)} \qquad \text{(A.1a)}$$

$$c(t) = \frac{C(t)}{\left(\theta^{NSS} Y(t)\right)} \qquad \text{(A.1b)}$$

The GDP, Y(t), and the reserves, X(t), evolve according to the SDE's:

$$dY(t) = \mu_Y Y(t)dt + \sigma_Y Y(t)dB(t)\,,\ 0 < \mu_Y < r \qquad (2.3)$$

$$dX(t) = \left[rX(t) - C(t) + \theta^{NSS} Y(t)\right]dt \qquad (2.12)$$

$X(t) \geq 0$ An SDE for x(t):

We shall derive an SDE for x(t), for non sudden stop, using Ito's lemma:

$$dx = \frac{\partial x}{\partial t}dt + \frac{\partial x}{\partial X}dX + \frac{\partial x}{\partial Y}dY + \frac{1}{2}\frac{\partial^2 x}{\partial X^2}(dX)^2 + \frac{\partial^2 x}{\partial X \partial Y}dXdY + \frac{1}{2}\frac{\partial^2 x}{\partial Y^2}(dY)^2 \qquad \text{(A.2)}$$

Since $x(t) = \frac{X(t)}{\left(\theta^{NSS} Y(t)\right)}$ Then $\frac{\partial x}{\partial t} = 0$ $\qquad \frac{\partial x}{\partial X} = \frac{1}{\theta^{NSS} Y}$

$$\frac{\partial x}{\partial Y} = \frac{-\theta^{NSS} X}{\left(\theta^{NSS} Y\right)^2} = \frac{-X}{\theta^{NSS} Y^2} \qquad \frac{\partial^2 x}{\partial X^2} = 0 \qquad \frac{\partial^2 x}{\partial X \partial Y} = \frac{-1}{\theta^{NSS} Y^2}$$

$$\frac{\partial^2 x}{\partial Y^2} = \frac{2X}{\theta^{NSS} Y^3} \qquad (dY)^2 = \sigma_Y^2 Y^2 dt \qquad (dX)^2 = 0 \qquad dXdY = 0$$

Substituting for the partial derivatives in equation (A.2) we get:

$$dx = \frac{1}{\theta^{NSS} Y}dX - \frac{X}{\theta^{NSS} Y^2}dY - \frac{1}{\theta^{NSS} Y^2}dXdY + \frac{X}{\theta^{NSS} Y^3}(dY)^2$$

Substituting for the complete derivatives we get:

$$dx = \frac{1}{\theta^{NSS}Y}\left[rX(t) - C(t) + \theta^{NSS}Y(t)\right]dt - \frac{X}{\theta^{NSS}Y^2}\left[\mu_Y Y(t)dt + \sigma_Y Y(t)dB(t)\right] + \frac{X}{\theta^{NSS}Y^3}\sigma_Y^2 Y^2 dt$$

Which is reduced to:

$$dx = \frac{X}{\theta^{NSS}Y(t)}\left[r - \frac{C(t)}{X(t)} + \frac{\theta^{NSS}Y(t)}{X(t)}\right]dt - \frac{X(t)}{\theta^{NSS}Y(t)}\left[\mu_Y dt + \sigma_Y dB(t)\right] + \frac{X(t)}{\theta^{NSS}Y(t)}\sigma_Y^2 dt$$

Rearrange we get:

$$dx = x(t)\left[r - \frac{C(t)}{X(t)} + \frac{\theta^{NSS}Y(t)}{X(t)}\right]dt - x(t)\left[\mu_Y dt + \sigma_Y dB(t)\right] + x(t)\sigma_Y^2 dt$$

$$= x(t)\left[\left(r - \mu_Y + \sigma_Y^2\right) - \frac{C(t)}{X(t)} + \frac{\theta^{NSS}Y(t)}{X(t)}\right]dt - x(t)\sigma_Y dB(t)$$

$$= x(t)\left[\left(r - \mu_Y + \sigma_Y^2\right) - \frac{c(t)}{x(t)} + \frac{1}{x(t)}\right]dt - x(t)\sigma_Y dB(t) \qquad \text{(A.3)}$$

Equation (A.3) is the desired SDE that describes the evolution of the normalized reserves x(t).

Exact Solution for the Normalized variables:

We shall now derive an exact ODE for the control, c(x), this is because all the components are explicitly independent of time and the diffusion is independent of the control (Abutaleb and Papaioannou 2009; Mangel 1985). The SDE describing the normalized reserves is given as:

$$dx(t) == x(t)\left[\left(r - \mu_Y + \sigma_Y^2\right) - \frac{c(t)}{x(t)} + \frac{1}{x(t)}\right]dt - x(t)\sigma_Y dB(t) \qquad \text{(A.3)}$$

The normalized utility function is given as:

$$U\left(c(x)\right) = \frac{c^{1-\gamma}}{1-\gamma} \qquad \text{(A.4)}$$

The objective function is given as:

$$V\left(x(t)\right) = \max_{c(s)} E\left\{\int_{t_0}^{\tau^{SS}} U(c(s))ds + \varphi_1\left(x(\tau^{SS})\right)\right\} \qquad \text{(A.5)}$$

Where all the variables involved are the normalized variables and where we have eliminated, for simplifications, the discount factor $e^{-r(s-t_0)}$.

The optimal normalized consumption c(t) satisfies the ordinary differential equation (ODE)[13]:

$$\frac{dc}{dx}\left[\frac{dF^u/dc}{db/dc}\frac{d^2b}{dc^2}-\frac{d^2F^u}{dc^2}\right]=\frac{2}{\sigma(x)}\left[\frac{b\,dF^u/dc}{db/dc}-F^u\right]+\frac{d^2F^u}{dcdx}-\frac{dF^u/dc}{db/dc}\frac{d^2b}{dcdx} \quad \text{(A.6)}$$

where u(x)=c(x)

$$F^u(x)=\frac{c^{1-\gamma}}{1-\gamma} \qquad b(x,c)=x\left[\left(r-\mu_Y+\sigma_Y^2\right)-\frac{c}{x}+\frac{1}{x}\right] \qquad \sigma(x)=-x\sigma_Y$$

We need the following derivatives:

$$\frac{dF^u(x)}{dc}=c^{-\gamma} \qquad \frac{d^2F^u(x)}{dc^2}=-\gamma\,c^{-\gamma-1} \qquad \frac{d^2F^u(x)}{dcdx}=0$$

$$\frac{db(x,c)}{dc}=-1 \qquad \frac{d^2b(x,c)}{dc^2}=0 \qquad \frac{d^2b(x,c)}{dcdx}=0$$

Substituting the different elements in equation (A.6) we get:

$$\frac{dc}{dx}\left[\gamma\,c^{-\gamma-1}\right]=-\frac{2}{x\sigma_Y}\left[-x\left[\left(r-\mu_Y+\sigma_Y^2\right)-\frac{c}{x}+\frac{1}{x}\right]c^{-\gamma}-\frac{c^{1-\gamma}}{1-\gamma}\right]$$

Dividing both sides by $c^{-\gamma}$ and rearrange we get:

$$\begin{aligned}\frac{dc}{dx}&=\frac{2c}{x\sigma_Y\gamma}\left[x\left[\left(r-\mu_Y+\sigma_Y^2\right)-\frac{c}{x}+\frac{1}{x}\right]+\frac{c}{1-\gamma}\right]\\&=\frac{2c}{\sigma_Y\gamma}\left[\left(r-\mu_Y+\sigma_Y^2\right)-\frac{c}{x}+\frac{1}{x}\right]+\frac{2c^2}{x\sigma_Y\gamma(1-\gamma)}\\&=\frac{2c}{\sigma_Y\gamma}\left[\left(r-\mu_Y+\sigma_Y^2\right)-\frac{c}{x}+\frac{1}{x}+\frac{c}{x(1-\gamma)}\right]\\&=\frac{2c}{\sigma_Y\gamma}\left[\left(r-\mu_Y+\sigma_Y^2\right)+\frac{-c(1-\gamma)+(1-\gamma)+c}{x(1-\gamma)}\right]\\&=\frac{2c}{\sigma_Y\gamma}\left[\left(r-\mu_Y+\sigma_Y^2\right)+\frac{-\gamma c+(1-\gamma)}{x(1-\gamma)}\right]\end{aligned}$$

[13] Abutaleb and Papaioannou (2009).

$$= \frac{2c}{\sigma_Y \gamma}\left[\left(r - \mu_Y + \sigma_Y^2\right) + \frac{\gamma c + (\gamma - 1)}{x(\gamma - 1)}\right] \quad \text{(A.7)}$$

This is the desired ODE for the optimal normalized consumption c(x).
Some approximations:

We know that c is in the order of 1, while x is in the order 0.2, and assuming that $\gamma >> 1$, then

$$\frac{dc}{dx} \approx \frac{2c}{\sigma_Y \gamma}\left(\frac{c+1}{x}\right)$$

Separating variables we get:

$$\frac{dc}{c(c+1)} \approx \frac{2}{\sigma_Y \gamma}\left(\frac{dx}{x}\right)$$

which has the solution:

$$\int \frac{dc}{c(c+1)} = \ln c - \ln(c+1) + \ln K = \ln \frac{Kc}{c+1} \approx \frac{2}{\sigma_Y \gamma} \ln x$$

where K is the constant of integration, and is determined from the boundary conditions.

i.e. $\frac{Kc}{c+1} \approx x^{\frac{2}{\sigma_Y \gamma}}$ Thus $c = \frac{x^{\frac{2}{\sigma_Y \gamma}}}{K - x^{\frac{2}{\sigma_Y \gamma}}}$,

A better formula is $c = K \frac{x^{\frac{2}{\sigma_Y \gamma}}}{1 - x^{\frac{2}{\sigma_Y \gamma}}}$ (A.8)

Substitute in the normalized reserves equation we get:

$$dx(t) = x(t)\left[\left(r - \mu_Y + \sigma_Y^2\right) - \frac{Kx(t)^{\frac{2}{\sigma_Y \gamma}}}{x(t)\left(1 - x(t)^{\frac{2}{\sigma_Y \gamma}}\right)} + \frac{1}{x(t)}\right]dt - x(t)\sigma_Y dB(t) \quad \text{(A.9)}$$

Appendix B: Optimal Consumptions and Optimal Values Invested in Risky Assets to Maximize the Utility Function

The risky asset's payoff process, F(t), is defined as:

$$dF(t) = r_F F(t)dt + F(t)JdN(t) \qquad \text{(B.1)}$$

Where r_F is the interest rate and will be assumed to be the same as r (the risk free interest rate), N(t) is a Poisson process, J is a random variable that has lognormal distribution. The probability density function of J, $f(J)$,is given as:

$$f(J) = \frac{1}{J\sqrt{2\pi\sigma_J^2}} \exp\frac{-(\ln J - \mu_J)^2}{2\sigma_J^2}$$

With $E\{J\} = \exp\left(\mu_J + \frac{\sigma_J^2}{2}\right)$

$$E\left\{J^2 - E\{J\}^2\right\} = \exp\left(2\mu_J + \sigma_J^2\right)\left[\exp\sigma_J^2 - 1\right]$$

The peak of J is located at $\exp\left(\mu_J - \sigma_J^2\right)$.

The evolution of reserves, X(t), becomes:

$$dX(t) = \left[r\left(X(t) - \xi(t)F(t)\right) - C(t) + A(t)\right]dt + \xi(t)dF(t) \qquad \text{(B.2)}$$

where C(t) is the consumption, $A(t) = \theta^{NSS}Y(t)$, $\theta^{NSS} \approx 1.2$, Y(t) is the GDP, $\xi(t)$ is the amount (or more precisely the number of units) invested in the risky assets with payoff F(t). Substitute the expression for the payoff of the risky assets in the reserves equation we get:

$$dX(t) = \left[r\left(X(t) - \xi(t)F(t)\right) - C(t) + A(t) + r\xi(t)F(t)\right]dt + \xi(t)F(t)JdN(t)$$

This equation is reduced to:

$$dX(t) = \left[rX(t) - C(t) + A(t)\right]dt + \xi(t)F(t)JdN(t) \qquad \text{(B.3)}$$

Working with normalized variables we get:

$$dx(t) = \left[\left(r - \mu_Y + \sigma_Y^2\right)x(t) + 1 + (r - r_F)\bar{\xi}(t) - c(t)\right]dt - \sigma_Y x(t)dB(t) + \bar{\xi}(t)E_J\{J\}dN(t) \quad \text{(C.7)}$$

The optimization problem now involves a portfolio decision. This decision is straightforward in the SS region since investing in the risky assets with payoff *F*(t) only means adding risk to the country's reserve holdings, without reward in terms of hedging value. Thus, the country picks $\xi(t)$ =0 in the SS region. The portfolio decision is interesting only in the non-sudden-stop (NSS) regions where the country is preparing itself for a potential sudden stop and we have $\xi(t) > 0$.

Notice that the quantity F(t) does not appear separately. It appears in the variable $\bar{\xi}(t)$. And since $\bar{\xi}(t)$ is actually a control variable that we need to estimate in order to maximize the utility function, we do not need the SDE for F(t) to be involved in the analysis.

We now solve the optimization problem in the following steps:
(1)The system dynamics becomes:

$$dx(t) = \left[\left(r - \mu_Y + \sigma_Y^2\right)x(t) + 1 + (r - r_F)\bar{\xi}(t) - c(t)\right]dt - \sigma_Y x(t)dB(t) + \bar{\xi}(t)E_J\{J\}dN(t) \quad \text{(C.7)}$$

The control variables are now $u_1(t) = c(t) = \dfrac{C(t)}{\theta^{NSS}Y(t)}$ and $u_2(t) = \bar{\xi}(t) = \dfrac{\xi(t)F(t)}{\theta^{NSS}Y(t)}$. The objective function in terms of the language of stochastic control is given as:

$$J^{\underline{u}}(t_0, x_0) = E\left(\int_{t_0}^{t_1} F^{\underline{u}}(s,x)ds + \varphi\left[x(t_1)\right] / x(t_0) = x_0\right) \quad \text{(B.4)}$$

where $F^{\underline{u}}(s,x) = e^{-r(s-t_0)}U\left(c(s)\right)$,

$$U\left(C\right) = \frac{c^{1-\gamma}}{1-\gamma},\ t_1 = \tau^{SS},\ \varphi\left[x(t_1)\right] = \text{terminal value conditions} \quad \text{(B.5)}$$

Define $\Psi(t,x) = \sup_{\underline{u}} J^{\underline{u}}(t,x)$ (B.6)

In a compact form, the system dynamics are given as:

$$dx(t) = b(x,\underline{u})dt + \sigma(x)dW(t) + h(\underline{u})dN(t) \qquad \text{(B.7)}$$

Where $b(x,\underline{u}) = \left[\left(r - \mu_Y + \sigma_Y^2\right)x(t) + 1 + (r - r_F)\bar{\xi}(t) - c(t)\right]$

$\sigma(x) = -\sigma_Y x(t)$, which is independent of the controls.

$h(\underline{u}) = \bar{\xi}(t)E_J\{J\}$

(2)The HJB equation becomes:

$$0 = \frac{\partial\Psi(s,x)}{\partial s} + \sup_{\underline{u}}\left\{F^{\underline{u}}(s,x) + b(x,\underline{u})\frac{\partial\Psi(s,x)}{\partial x} + \frac{1}{2}\sigma^2(x)\frac{\partial^2\Psi(s,x)}{\partial x^2} + \lambda\left[\Psi(s,x+h(\underline{u})) - \Psi(s,x)\right]\right\}$$

(B.8)

The shift term "$\left[\Psi(s,x+h(\underline{u})) - \Psi(s,x)\right]$", due to the Poisson process, will cause many mathematical difficulties. Thus, we shall try to approximate this term.

In what to follow one will be able to find an approximate analytic solution if we further assume that:

$$\Psi(s,x) = g(s)e^{\alpha x} \qquad \text{(B.9)}$$

Which yields

$$\frac{\partial\Psi(s,x)}{\partial s} = \frac{\partial g(s)}{\partial s}e^{\alpha x} \qquad \frac{\partial\Psi(s,x)}{\partial x} = \alpha g(s)e^{\alpha x} \qquad \frac{\partial^2\Psi(s,x)}{\partial x^2} = \alpha^2 g(s)e^{\alpha x}$$

Then $\Psi(s,x+h(\underline{u})) = g(s)e^{\alpha\left(x+h(\underline{u})\right)} \qquad = g(s)e^{\alpha\left(x+\bar{\xi}(s)E_J\{J\}\right)}$

$$= \Psi(s,x)e^{\alpha\left(\bar{\xi}(s)E_J\{J\}\right)} \qquad \text{(B.10)}$$

where $\bar{\xi} = \dfrac{\xi F}{\left(\theta^{NSS}Y\right)}$

Substitute we get:

$$\lambda\left[\Psi(s,x+h(\underline{u})) - \Psi(s,x)\right] = \lambda\Psi(s,x)\left[e^{\alpha\left(\bar{\xi}(s)E_J\{J\}\right)} - 1\right]$$

$$= \lambda g(s)e^{\alpha x}\left[e^{\alpha\left(\bar{\xi}(s)E_J\{J\}\right)} - 1\right] \qquad \text{(B.11)}$$

We need to find the different elements of the HJB equation (B.8). We have:

$$\sigma^2(x)\frac{\partial^2\Psi(s,x)}{\partial x^2} = \sigma_Y^2 x^2(s)\alpha^2 g(s)e^{\alpha x} \tag{B.12}$$

$$b(x,\underline{u})\frac{\partial\Psi(s,x)}{\partial x} = \left[\left(r-\mu_Y+\sigma_Y^2\right)x(s)+1+(r-r_F)\bar{\xi}(s)-c(s)\right]\frac{\partial g(s)}{\partial s}e^{\alpha x} \tag{B.13}$$

We need to find the optimal controls and the solution of the HJB equation.

(3)Substitute the above derivatives in the different elements of the HJB equation and find the optimal controls:

$$\sup_{\underline{u}}\left\{F^{\underline{u}}(s,x)+b(x,\underline{u})\frac{\partial\Psi(s,x)}{\partial x}+\frac{1}{2}\sigma^2(x)\frac{\partial^2\Psi(s,x)}{\partial x^2}+\lambda\left[\Psi(s,x+h(\underline{u}))-\Psi(s,x)\right]\right\}=$$

$$+\frac{1}{2}\sigma^2(x)\frac{\partial^2\Psi(s,x)}{\partial x^2}+\sup_{\underline{u}}\left\{F^{\underline{u}}(s,x)+b(x,\underline{u})\frac{\partial\Psi(s,x)}{\partial x}+\lambda\left[\Psi(s,x+h(\underline{u}))-\Psi(s,x)\right]\right\}$$

(B.14) And

$$\sup_{\underline{u}}\left\{F^{\underline{u}}(s,x)+b(x,\underline{u})\frac{\partial\Psi(s,x)}{\partial x}+\lambda\left[\Psi(s,x+h(\underline{u}))-\Psi(s,x)\right]\right\}$$

$$=\sup_{\underline{u}}\left\{e^{-r(s-t_0)}\frac{c^{1-\gamma}}{1-\gamma}+\left[\left(r-\mu_Y+\sigma_Y^2\right)x+1+(r-r_F)\bar{\xi}-c\right]\frac{\partial\Psi(s,x)}{\partial x}+\lambda\Psi(s,x)\left[e^{\alpha\left(\bar{\xi}E_J\{J\}\right)}-1\right]\right\}$$

(B.15)

Maximizing with respect to c(s) we get:

$$e^{-r(s-t_0)}c(s)^{-\gamma}-\frac{\partial\Psi(s,x)}{\partial x}=0$$

1.e. $c(s)=\left[e^{r(s-t_0)}\frac{\partial\Psi(s,x)}{\partial x}\right]^{\left(\frac{-1}{\gamma}\right)}$ (B.16)

This is an expression for the optimal value of the consumption that maximizes the utility function.

Maximizing with respect to $\bar{\xi}$ we get:

$$(r - r_F) + \alpha E_J\{J\}\lambda\Psi(s,x)e^{\alpha(\bar{\xi}E_J\{J\})} = 0$$

Rearrange we get:

$$\lambda\Psi(s,x)e^{\alpha(\bar{\xi}E_J\{J\})} = \frac{-(r - r_F)e}{\alpha E_J\{J\}}$$

1.e. $$\bar{\xi}(s) = \frac{1}{\alpha E_J\{J\}}\ln\left[\frac{-(r - r_F)}{\alpha E_J\{J\}\lambda\Psi(s,x)}\right] \qquad \text{(B.17)}$$

Substitute the derived optimal values in the supremum function we get:

$$\sup_{\underline{u}}\left\{e^{-r(s-t_0)}\frac{c^{1-\gamma}}{1-\gamma} + \left[\left(r - \mu_Y + \sigma_Y^2\right)x + 1 + (r - r_F)\bar{\xi} - c\right]\frac{\partial\Psi(s,x)}{\partial x} + \lambda\Psi(s,x)\left[e^{\alpha(\bar{\xi}E_J\{J\})} - 1\right]\right\}$$

$$= e^{-r(s-t_0)}\frac{\left(e^{r(s-t)}\frac{\partial\Psi(s,x)}{\partial x}\right)^{\frac{\gamma-1}{\gamma}}}{1-\gamma} + \left[\left(r - \mu_Y + \sigma_Y^2\right)x + 1\right]\frac{\partial\Psi(s,x)}{\partial x}$$

$$+ (r - r_F)\bar{\xi}\frac{\partial\Psi(s,x)}{\partial x} - \left(e^{r(s-t)}\frac{\partial\Psi(s,x)}{\partial x}\right)^{\left(\frac{-1}{\gamma}\right)}\frac{\partial\Psi(s,x)}{\partial x} + \frac{-(r - r_F)e}{\alpha E_J\{J\}} - \lambda\Psi(s,x)$$

(B.18)

For large values of γ, and for small values of $(r - r_F)$, we get the approximation:

$$\sup_{\underline{u}}\left\{e^{-r(s-t_0)}\frac{c^{1-\gamma}}{1-\gamma} + \left[\left(r - \mu_Y + \sigma_Y^2\right)x + 1 + (r - r_F)\bar{\xi} - c\right]\frac{\partial\Psi(s,x)}{\partial x} + \lambda\Psi(s,x)\left[e^{\alpha(\bar{\xi}E_J\{J\})} - 1\right]\right\}$$

$$\approx \frac{1}{1-\gamma}\frac{\partial\Psi(s,x)}{\partial x} + \left[\left(r - \mu_Y + \sigma_Y^2\right)x + 1\right]\frac{\partial\Psi(s,x)}{\partial x} - e^{\frac{-r}{\gamma}(s-t_0)}\frac{\partial\Psi(s,x)}{\partial x} - \lambda\Psi(s,x)$$

(B.19)

(4)Solve the HJB equation to find $\Psi(s,x)$.

Substitute the different derived elements in the HJB equation we get:

$$0=\frac{\partial\Psi(s,x)}{\partial s}+\sup_{\underline{u}}\left\{F^{\underline{u}}(s,x)+b(x,\underline{u})\frac{\partial\Psi(s,x)}{\partial x}+\frac{1}{2}\sigma^2(x)\frac{\partial^2\Psi(s,x)}{\partial x^2}+\lambda\left[\Psi(s,x+h(\underline{u}))-\Psi(s,x)\right]\right\}$$

$$\approx\frac{\partial\Psi(s,x)}{\partial s}+\frac{1}{1-\gamma}\frac{\partial\Psi(s,x)}{\partial x}+\left[\left(r-\mu_Y+\sigma_Y^2\right)x+1\right]\frac{\partial\Psi(s,x)}{\partial x}-e^{\frac{-r}{\gamma}(s-t_0)}\frac{\partial\Psi(s,x)}{\partial x}-\lambda\Psi(s,x)$$

$$+\frac{1}{2}\sigma_Y^2x^2(s)\frac{\partial^2\Psi(s,x)}{\partial x^2}$$

(B.20)

which is reduced to:

$$0\approx\frac{\partial\Psi(s,x)}{\partial s}+\left[\frac{1}{1-\gamma}+\left(r-\mu_Y+\sigma_Y^2\right)x+1-e^{\frac{-r}{\gamma}(s-t_0)}\right]\frac{\partial\Psi(s,x)}{\partial x}-\lambda\Psi(s,x)+\frac{1}{2}\sigma_Y^2x^2(s)\frac{\partial^2\Psi(s,x)}{\partial x^2}$$

(B.21)

Substitute $\frac{\partial\Psi(s,x)}{\partial s}=\frac{\partial g(s)}{\partial s}e^{\alpha x}$, $\frac{\partial\Psi(s,x)}{\partial x}=\alpha g(s)e^{\alpha x}$, $\frac{\partial^2\Psi(s,x)}{\partial x^2}=\alpha^2g(s)e^{\alpha x}$, in the HJB equation we get:

$$0\approx\frac{\partial g(s)}{\partial s}e^{\alpha x}+\left[\frac{1}{1-\gamma}+\left(r-\mu_Y+\sigma_Y^2\right)x+1-e^{\frac{-r}{\gamma}(s-t_0)}\right]\left(\alpha g(s)e^{\alpha x}\right)-\lambda g(s)e^{\alpha x}$$

$$+\frac{1}{2}\sigma_Y^2x^2(s)\left(\alpha^2g(s)e^{\alpha x}\right)$$

Dividing by $e^{\alpha x}$, we get:

$$0\approx\frac{\partial g(s)}{\partial s}+\left[\frac{1}{1-\gamma}+\left(r-\mu_Y+\sigma_Y^2\right)x+1-e^{\frac{-r}{\gamma}(s-t_0)}\right]\left(\alpha g(s)\right)-\lambda g(s)+\frac{1}{2}\sigma_Y^2x^2(s)\left(\alpha^2g(s)\right)$$

(B.22) Noting that x(s) is around 0.1, σ_Y^2 is in the order of 10^{-2}, we get:

$$0\approx\frac{\partial g(s)}{\partial s}-\lambda g(s)\qquad\text{(B.23)}$$

Which yields:

$$g(s) = K_g \exp \lambda s \tag{B.24}$$

where K_g is a constant of integration and is determined from the boundary values.

Thus, the optimal objective function is approximately given as:

$$\Psi(s,x) = g(s)e^{\alpha x} \quad = K_g e^{\lambda s} e^{\alpha x} \tag{B.25}$$

Substitute in the expressions of the controllers we get the approximate values:

$$\begin{aligned} c(s) = \frac{C(s)}{\theta^{NSS} Y(s)} &\approx \left[e^{r(s-t_0)} \alpha K_g e^{\lambda s} e^{\alpha x(s)} \right]^{\left(\frac{-1}{\gamma}\right)} \\ &\approx \left(\alpha K_g\right)^{\left(\frac{-1}{\gamma}\right)} \exp\left[\left(\frac{-1}{\gamma}\right) \left(r(s-t_0) + \lambda s + \alpha x(s) \right) \right] \end{aligned} \tag{B.26}$$

Thus, for real value of "$\left(\alpha K_g\right)^{\left(\frac{-1}{\gamma}\right)}$", $\left(\alpha K_g\right) > 0$.

$$\begin{aligned} \bar{\xi}(s) = \frac{\xi(s) F(s)}{\theta^{NSS} Y(s)} &\approx \frac{1}{\alpha E_J \{J\}} \ln\left[\frac{-(r - r_F)}{\alpha E_J \{J\} \lambda K_g e^{\lambda s} e^{\alpha x}} \right] \\ &\approx \frac{1}{\alpha E_J \{J\}} \left\{ \ln\left[\frac{(r_F - r)}{\alpha E_J \{J\} \lambda K_g} \right] - \ln e^{(\lambda s + \alpha x(s))} \right\} \\ &\approx \frac{1}{\alpha E_J \{J\}} \left\{ \ln\left[\frac{(r_F - r)}{\alpha E_J \{J\} \lambda K_g} \right] - \left[\lambda s + \alpha x(s) \right] \right\} \end{aligned} \tag{B.27}$$

Appendix C: Ito lemma for Jump diffusion process[14]

Assume the SDE governing the system is given as:

$$d\underline{X}(t) = \underline{f}(\underline{X}(t),t)dt + g(\underline{X}(t),t)d\underline{B}(t) + h(\underline{X}(t),\underline{q},t)d\underline{P}(\underline{X}(t),t) \quad \text{(C.1)}$$

Where $\underline{q}$ is the vector of the random amplitudes of the Poisson vector process $\underline{P}(\underline{X}(t),t)$. Assume that we have a continuous function $\Pi(\underline{X}(t),t)$ and we need to find its SDE. Using Ito lemma for diffusion jump process we get the general expression:

$$d\Pi(\underline{X}(t),t) = \frac{\partial \Pi}{\partial t}dt + \sum_i \frac{\partial \Pi}{\partial X_i}\left(f_i dt + \sum_k g_{i,k} dB_k \right) + \frac{1}{2}\sum_i \sum_j \sum_k \sum_l \frac{\partial^2 \Pi}{\partial X_i \partial X_j} g_{i,k} g_{j,l} dB_k dB_l$$
$$+ \sum_j E_{q_j}\left\{ \Pi(\underline{X}(t) + \underline{h}_j, t) - \Pi(\underline{X}(t),t) \right\} dP_j$$

(C.2)

where $\underline{h}_j$ is the jth column of the matrix $h(\underline{X}(t),\underline{q},t)$.

We apply this lemma to the system under study where now:

$$\underline{X}(t) = \begin{bmatrix} X(t) \\ Y(t) \\ F(t) \end{bmatrix}, \ \Pi(\underline{X}(t),t) = \frac{X(t)}{\theta^{NSS} Y(t)} = x(t) \qquad \text{(C.3)}$$

The SDE describing the evolution of the economy is given as:

$$\begin{bmatrix} dX(t) \\ dY(t) \\ dF(t) \end{bmatrix} = \left\{ \begin{bmatrix} r & \theta^{NSS} & (r - r_F)\xi(t) \\ 0 & \mu_Y & 0 \\ 0 & 0 & r_F \end{bmatrix} \begin{bmatrix} X(t) \\ Y(t) \\ F(t) \end{bmatrix} + \begin{bmatrix} -C(t) \\ 0 \\ 0 \end{bmatrix} \right\} dt + \begin{bmatrix} 0 \\ \sigma_Y Y(t) \\ 0 \end{bmatrix} dB(t) + \begin{bmatrix} \xi(t)F(t)J \\ 0 \\ F(t)J \end{bmatrix} dN(t)$$

(3.4)

[14] Hanson (2007).

Thus,

$$d\Pi(\underline{X}(t),t) = \frac{\partial \Pi}{\partial t}dt + \sum_i \frac{\partial \Pi}{\partial X_i}\left(f_i dt + g_i dB\right) + \frac{1}{2}\sum_i\sum_j \frac{\partial^2 \Pi}{\partial X_i \partial X_j} g_i g_j dt \qquad \text{(C.4)}$$
$$+ E_J\{\Pi(\underline{X}(t)+\underline{h},t) - \Pi(\underline{X}(t),t)\}dN$$

where

$$f_1 = rX(t) + \theta^{NSS}Y(t) + (r-r_F)\xi(t)F(t) - C(t) = rX_1(t) + \theta^{NSS}X_2(t) + \left(r-r_F\right)\xi(t)X_3(t) - C(t)$$

$f_2 = \mu_Y Y(t) = \mu_Y X_2(t)$, $f_3 = r_F F(t) = r_F X_3(t)$, $g_1 = 0$, $g_2 = \sigma_Y Y(t) = \sigma_Y X_2(t)$, $g_3 = 0$, $h_1 = \xi(t)F(t)J = \xi(t)X_3(t)J$, $h_2 = 0$, $h_3 = F(t)J = X_3(t)J$.

We substitute these values to get the different elements of the transformed system SDE.

$$\Pi(\underline{X}(t)+\underline{h},t) - \Pi(\underline{X}(t),t) = \frac{X(t)+\xi(t)F(t)J}{\theta^{NSS}Y(t)} - \frac{X(t)}{\theta^{NSS}Y(t)}$$
$$= \frac{\xi(t)F(t)J}{\theta^{NSS}Y(t)} \quad = \bar{\xi}(t)J \qquad \text{(C.5)}$$

Thus, $E_J\{\Pi(\underline{X}(t)+\underline{h},t) - \Pi(\underline{X}(t),t)\} = \bar{\xi}(t)E_J\{J\}$ (C.6)

$$\frac{\partial \Pi}{\partial t} = 0 \qquad \frac{\partial \Pi}{\partial X_1} = \frac{1}{\theta^{NSS}Y(t)}, \qquad \frac{\partial^2 \Pi}{\partial X_1^2} = 0, \qquad \frac{\partial^2 \Pi}{\partial X_1 \partial X_2} = \frac{-1}{\theta^{NSS}Y^2(t)},$$
$$\frac{\partial^2 \Pi}{\partial X_1 \partial X_3} = 0 \qquad \frac{\partial \Pi}{\partial X_2} = \frac{-X(t)}{\theta^{NSS}Y^2(t)}, \quad \frac{\partial^2 \Pi}{\partial X_2^2} = \frac{2X(t)}{\theta^{NSS}Y^3(t)}, \quad \frac{\partial^2 \Pi}{\partial X_2 \partial X_3} = 0$$
$$\frac{\partial \Pi}{\partial X_3} = 0$$

Substituting in the general form of the system SDE we get:

$$d\Pi(\underline{X}(t),t) = \frac{1}{\theta^{NSS}Y(t)}\left[rX(t) + \theta^{NSS}Y(t) + (r-r_F)\xi(t)F(t) - C(t)\right]dt + \frac{-X(t)}{\theta^{NSS}Y^2(t)}\mu_Y Y(t)dt$$
$$+ \frac{-X(t)}{\theta^{NSS}Y^2(t)}\sigma_Y Y(t)dB(t) + \frac{X(t)}{\theta^{NSS}Y^3(t)}\sigma_Y^2 Y^2(t)dt + \bar{\xi}(t)E_J\{J\}dN(t)$$

Collecting terms and rearranging we get:

$$dx(t) = \left[\left(r - \mu_Y + \sigma_Y^2\right)x(t) + 1 + (r - r_F)\bar{\xi}(t) - c(t)\right]dt - \sigma_Y x(t)dB(t) + \bar{\xi}(t)E_J\{J\}dN(t)$$
(C.7)

Which is an SDE in the normalized reserves with two controllers, c(t) and $\bar{\xi}(t)$.

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ANNEX

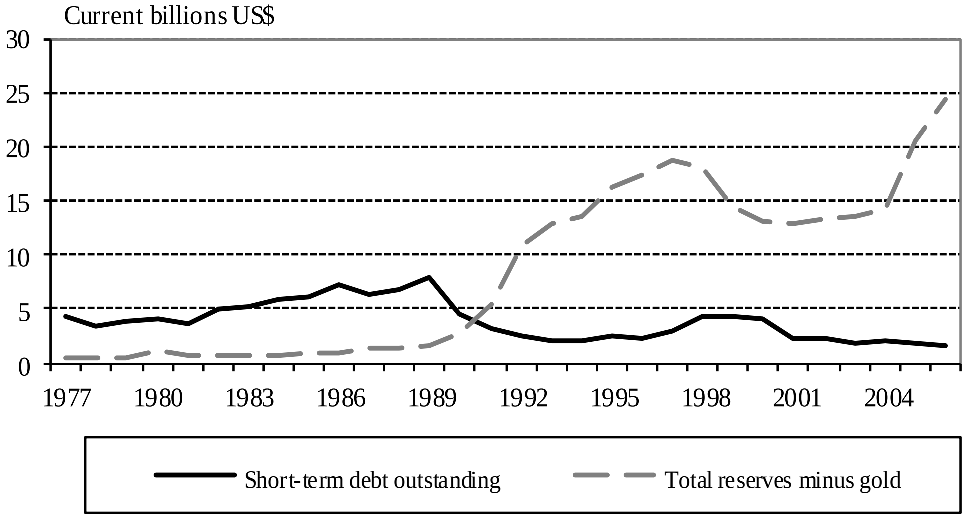
Table 1. Market Estimates of Assets Under Management for SWFs Based on Latest Available Information (As of February 2008)

Name of Fund	
1. Oil and Gas Exporting Countries	
UAE	Abu Dhabi Investment Authority, (1976[1])
Norway	Government Pension Fund-Global
Saudi Arabia1/	No designated name
Kuwait 1/	Reserve Fund for the Future Generations
	Government Reserve Fund
Russia 1/	Reserve Fund
	National Welfare Fund
Libya	Libyan Investment Authority
Qatar	State Reserve Fund/Stabilization fund
Algeria	Reserve Fund/Revenue Regulation Fund
USA (Alaska)	Alaska Permanent Reserve Fund
Brunei	Brunei Investment Authority, (1983[2])
Kazakhstan	National Fund
Malaysia	Khazanah National BHD
Canada	Alberta Heritage Savings Trust Fund
Nigeria	Excess Crude Account
Iran	Foreign Exchange Reserve Fund, (1976[3])
Azerbaijan	State Oil Fund
Oman	State General Reserve Fund
Timor-Leste	Petroleum Fund of Timor-Leste
Venezuela	FIEM
Trinidad & Tobago	Revenue Stabilization Fund
Singapore 1/	Government Investment Corp.
	Temasek
China	State Foreign Exchange Investment Co.
Hong Kong	Investment Portfolio (HKMA)
Korea	Korea Investment Corp.
Taiwan, Province of China	National Stabilization Fund
Australia	Australian Future Fund
Chile	FEES and Pension Reserve Fund
Botswana 1/	Pula Fund
	Kiribati Revenue Equalization Fund
Total	

Source: IMF. 2008. Chile: Selected Issues Paper. IMF Country Report No. 08/239, July, Washington D.C.

[1, 2, 3] As cited in Blundell-Wignall, Hu, and Yermo (2008).

Figure 1. Total Reserves minus Gold and Short-term Debt Outstanding.



Source: WDI-Database 2008.