Journal of Economic Cooperation and Development, **41**, 2 (2020), 17-46

Redevelopment of the Matching Model: Decortications and Application on the Tunisian Labor Market

Monem Abidi[1]

The article proposes to build a new platform that traces more robust legibility between skilled and unskilled unemployment on the labor market, by dissecting the factors that influence the decision-making process regarding firms and employment people.

This article reviews the matching model, thus specifying a new formulation, focusing on the hiring probabilities and taking into account the depreciation of diplomas. This demarcation makes it possible to compare the probabilities of hiring and leaving unemployment of these two categories according to different parameters such as the tension on the labor market and the unemployment rate of graduates. Since then, we have been synthesizing the joint behavior of unemployed people and companies. This modeling allows a better understanding of the probability of exit from unemployment, determination of wages, utilities and equilibrium.

The calibration, of the decomposition that we have constructed from data relating to the Tunisian economy, allows us to understand the interactions between the different social partners in terms of tension on the labor market, as well as to weigh the impact of government profit transfer policy.

JEL: E24, E32, G12, J23.

Key-Words: Labor market; Matching; Salary; Skilled unemployment; Unskilled unemployment; Equilibrium.

1. Introduction

Unemployment is evolved and is transformed in depth. The classic definition is that being unemployed means being without work. With the

[1] VPNC Research Laboratory N-O, FSJEG of Jendouba, University of Jendouba, Tunisia. E-mail: abidi.monem93@gmail.com

emergence of industrial society, the notion of wages was incorporated, which in turn elevates the idea of marginal productivity. It is from this conception that this social class takes shape, and changes its status from unemployed to job seekers, it is in this sense that public institutions act to integrate these unemployed into the labor market.

The volume of unemployment is a key indicator of the state of health of the economy, and at the same time as a society. However, the volume of this mass is in most cases truncated, moreover, the borders which delimit unskilled unemployment, qualified and inactive unemployment are blurred imprecise, which puts question marks on the true number of unemployed.

It is difficult to prove that economic stagnation is solely responsible for unemployment and that unemployment is reduced simply by economic expansion (Davis and Haltiwanger, 1999). Consequently, the automatic correlation which links the reduction in unemployment to economic growth is no longer robust, and validation is unreal.

Unemployment is therefore not only the result of an economic situation, but also the origin and structural order. As much, the mode of operation and structuring of the labor market gives rise to a new macroeconomic conception of unemployment.

The matching theory is the most solid approach that attempts to analyze this new conception of structural unemployment, explaining that the resorption of unemployment can not be done in the presence of an inadequate matching process even in times of economic expansion, which results in the overwhelming of the procedure of matching supply and demand in the labor market.

Our analysis is focused on the matching model[2], where empirical evidence suggests that this phenomenon has become widespread in recent

[2] Assumptions adopted:

- The economy is assumed to be defined in continuous time with two benchmark goods (monetary goods and labor).
- Risk aversion for individuals (between unemployment and employment).
- The destruction of jobs is done according to a process which follows the law of parameter poisson « $\lambda > 0$ ».

years. We adapt and extend, therefore, the model presented by Pissarides (1999), and then we use different probabilities and combination of parameters to arrive at statistics that allow better visibility, that can be useful for the political recommendations as it gets closer, benefits and the overall balance of the economy.

Our article also relies in part on the job search theory, the corresponding literature that focuses on the mismatch of skills with bilateral heterogeneity refers to Pissarides (1994), Mortensen and Pissarides (1999), Shi (2002), Albrecht and Vroman (2002), Gautier (2002), Pries (2008), Dolado, Jansen and Jimeno (2009), Pissarides (2009), and Chassambolli (2011).

Albrecht and Vroman (2002)[3] consider that the labor market is composed of skilled and unskilled workers, on the other side of the posts which is differentiated by their skills needs, they contemplate, and that the distribution of skills of workers is exogenous, whereas the needs in terms of business skills are endogenous.

Thus, they manage to determine a balancing relationship between the skills of workers and jobs, unemployment and wages.

Dolado, Jansen, and Jimeno (2009) analyze this phenomenon of imbalance between skills, which requires employment and the level of education of workers by evoking the two principles, research on the job (OTJ) and job transitions to employment (JTJ), and they show that this decomposition has implications for the response of the labor market, in terms of changes in supply and demand for skills, unemployment, wages and salaries, and the job composition.

We also rely on other analyzes that deal with the matching model, such as Rogerson and Al (2005), Davidson and Al (2008) and Agense and Hromcov (2016), which integrate globalization as factors in Increased

---

- The active population is made up of individuals with an infinite lifespan and a constant "*k*" size.
- Constant income is distributed to the unemployed.
- Impairment of diplomas.

[3] This is similar to ours, but it does not take into account the probabilities of hiring and leaving skilled and unskilled unemployment, as well as the phenomenon of impairment of diplomas.

levels of productivity[4] to understand the reaction of less-skilled workers to globalization, as well as attitudes towards higher education.

Wang and Al (2008) show that higher-level workers with more general skills are more likely to benefit from work after retirement, based on the idea that high and more general skills are easily usable in low-skill sectors and also in specific sectors.

However, Oreopoulus, Van Wachter and Heisz (2012) show that the imbalance of overqualification can lead to long-term negative effects on the overall balance of the economy.

The formulation of the model begins with the determination of the framework that delimits this relationship between firms and the skilled and unskilled unemployed. In this reformulation, we try to answer a few questions related to the growth of unemployment among graduates and try to build a kind of possible balance between unskilled unemployment and qualified unemployment while respecting the framework of the theory of matching.

The article is organized as follows:

Section I deals with the general presentation of the model and the movements in and out of unemployment, distinguishing the qualified unemployed from the unskilled unemployed. This demarcation makes it possible to compare the probabilities of hiring and leaving unemployment of these two categories according to different parameters such as the tension on the labor market and the unemployment rate of graduates.

Section II examines the earnings functions of the various social partners, which will help to distinguish the parameters that guide decisions.

Section III discusses the impact of this modeling on the determination of equilibrium and introduces new factors that may affect equilibrium. It turns out that the decomposition of the unemployed is decisive concerning the tension in the labor market, the determination of wages, the creation and destruction of jobs, and the transfer of benefits.

---

[4] Made possible thanks to new technological advances embodied in the import.

Section IV completes the study with a digital application adopted in the Tunisian case, where the impact of the policy of transfer of benefits on the labor market will be discussed.

2. Movements of entry and output of unemployment[5]

The total number of unemployed persons for a given period is indicated by "*U*" and "*V*" the number of vacant posts. Note that "*U*" refers to the job supply while "*V*" represents the demand, and the probability that a vacant job will receive an offer is "$j = 1 / V$". At the same time, during this period the number of unemployed people likely to be in contact with a vacant job is a binomial variable, with the parameter "$j = 1 / V$" and "$b = U$". If we assume that the number of vacancies and unemployed is large, the variable which traces the hiring will follow the law of the poisson of the parameter "$bj = U / V$", thus the probability that a vacant job perceives a number "*a*" unemployed "*i*" is:

$$J^i(a) = \left(\frac{u^i}{v}\right)^a \frac{e^{\frac{-u^i}{v}}}{a!}\ , \text{« } i=q,\ n \text{ ».} \qquad \textbf{(1)}$$

With "u^i" represents the qualified and unskilled unemployment rate, while "*v*" represents the vacancy rate.

It is assumed that in the labor market a skilled unemployed worker is more in demand than an unskilled unemployed worker who is expected to be more productive.

2.1. Hiring probabilities

The instantaneous probability "p^q" of hiring a qualified unemployed person is $p^q = 1 - e^{\frac{-u^q}{v}}$ (**2**), this probability also expresses the probability of receiving an offer from at least one skilled unemployed person: $p^q = 1 - J^q(0)$ (**3**).

[5] All the demonstrations that come are specific to the author.

We note by $\delta = v/u$ [6] (***4***), the indicator of tension in the labor market.

The proportion of the qualified unemployed is defined as overall unemployment by:

$\beta u = u^q$ with $0 < \beta < 1$

From where: $p^q = 1 - e^{\frac{-\beta}{\delta}}$ ***(5)***.

$\lim_{\beta \to 0} p^q = 0$. This explains why when the proportion of qualified unemployed is low the chances of recruiting one will be very low.

The possibility of hiring an unskilled unemployed person is of instant probability "p^n", which can be calculated from the probability that describes that a vacant job does not receive an offer from the skilled unemployed category, and at least one offer from the unskilled unemployed category, which gives: $p^n = J^q(0)\left(1 - J^q(0)\right)$ [7]

So we can write:

$$\Rightarrow p^n = e^{\frac{-u^q}{v}}\left(1 - e^{\frac{-u^n}{v}}\right)$$ Knows that $v = \delta u$

$$\Rightarrow p^n = e^{\frac{-\beta u}{\delta u}}\left(1 - e^{\frac{-(1-\beta)u}{\delta u}}\right)$$

$$\Rightarrow p^n = e^{\frac{-\beta}{\delta}}\left(1 - e^{\frac{-(1-\beta)}{\delta}}\right) \qquad \textbf{\textit{(6)}}.$$

---

[6] Two voltage indicators could also be defined $\delta^i = \frac{v}{v^i}$ with $i = q, n$

With: $\delta^q + \delta^n = \frac{v}{\alpha(1-\alpha u)}$ which admits of solution only if $u \leq \frac{1}{2}$

[7] The proportion of unqualified unemployed will be: $u^n = (1-\beta)u$

The analytical contribution of these two equations (***5***) and (***6***) is manifested by the study of the derivative: $\frac{\partial p^q}{\partial \beta} \succ 0$, $\frac{\partial p^n}{\partial \beta} \prec 0$. This explains why once the proportion of qualified unemployed increases in the population, the probability of recruiting unskilled unemployed decreases.

$\frac{\partial p^q}{\partial \delta} \prec 0$: This means that if the tension in the labor market increases, the chance of recruiting qualified unemployed people decreases.

$\frac{\partial p^n}{\partial \delta} \prec 0, iff\ \beta \succ \delta$, brings back to write: $u^q \prec v$ [8], Similarly, if the tension on the labor market increases the probability of hiring an unskilled unemployed decreases.

The case $\frac{\partial p^n}{\partial \delta} \succ 0, \Rightarrow (\beta \prec \delta), so\ (u^q \succ v)$ explains that while the number of vacancies is relatively small compared to the number of qualified unemployed, the growth in the number of firms (at u^q given) increases the probability of recruiting unskilled unemployed.

We can summarize as much from (***5***) and (***6***) that the growth in unemployment of qualified individuals is no longer limited to the increase in their number, but essentially to the increase in tension on the labor market.

2.2.Probabilities of leaving unemployment

The deduction of the probability of hiring by each type of unemployment is made by multiplying the instant probability of hiring by the number of vacant positions (*V*). We can, therefore, note that the number of hires of qualified and unskilled unemployed is:

[8] See Appendix

$$P^i = p^i V$$

$$\Rightarrow P^q = \left(1 - e^{\frac{-\beta}{\delta}}\right)\left(\frac{\delta}{\beta}\right)u^q$$

And $P^n = e^{\frac{-\beta}{\delta}}\left(1 - e^{\frac{-(1-\beta)}{\delta}}\right)\left(\frac{\delta}{1-\beta}\right)u^n$

Since then, we can deduce the instantaneous probability of leaving unemployment (*J*), by dividing the number of hires (*P*) by the number of unemployed by category (u^i). $\Rightarrow J^q = \left(1 - e^{\frac{-\beta}{\delta}}\right)\left(\frac{\delta}{\beta}\right)$ And

$$J^n = e^{\frac{-\beta}{\delta}}\left(1 - e^{\frac{-(1-\beta)}{\delta}}\right)\left(\frac{\delta}{1-\beta}\right).$$

So: $\Rightarrow \frac{\partial J^q}{\partial \delta} = \left(1 - e^{\frac{-\beta}{\delta}}\left(1 + \frac{\beta}{\delta}\right)\right)\left(\frac{1}{\beta}\right) \succ 0$ [9] *(7)*

Likewise

$$\frac{\partial J^n}{\partial \delta} = \left(e^{\frac{-\beta}{\delta}}\left(1 + \frac{\beta}{\delta}\right) - e^{\frac{-1}{\delta}}\left(1 + \frac{1}{\delta}\right)\right)\left(\frac{1}{1-\beta}\right) \succ 0.$$

This shows that the instant probabilities of leaving unemployment for the skilled and unskilled increase if the tension in the labor market increases, which is explained by the increase in vacancies (we suppose that for a period unemployment is constant).

Comparing the variation in the instant probability of hiring skilled workers« p^q » ($\frac{\partial p^q}{\partial \delta} \prec 0$) with that of the instant probability

[9] Since $e^{-u}(1+u)<1$.

of leaving unemployment «J^q» ($\frac{\partial J^q}{\partial \delta} \succ 0$). Shows a variation in the opposite direction compared to the tension on the labor market. Which evokes a conflict of objectives between the firms and the qualified unemployed (the same case occurs $\frac{\partial p^n}{\partial \delta} \prec 0$ and $\frac{\partial J^n}{\partial \lambda \delta} \succ 0$).

The case study of the unskilled unemployed presents no problem, so $\frac{\partial p^n}{\partial \delta} \succ 0$ and $\frac{\partial J^n}{\partial \delta} \succ 0$, The analysis of the variation in terms of tension on the labor market explains a consistency between the instant probability of hiring the unskilled, and that of their exit from unemployment.

Analysis of the variation compared to the fraction of the skilled unemployed in overall unemployment gives:

$$\frac{\partial J^q}{\partial \beta} = \left(e^{\frac{-\beta}{\delta}} \left(1 + \frac{\beta}{\delta} \right) - 1 \right) \left(\frac{\delta}{\beta^2} \right) \prec 0$$, according to *(7)*, likewise $\frac{\partial J^n}{\partial \beta} \succ 0$

, thus the instantaneous probability of the exit from unemployment is even lower than the fraction of the category concerned in the overall unemployment is large. Explaining that if the number of unskilled unemployed increases, the chances of people in this category leaving unemployment decreases, and those of the skilled unemployed increase.

We also note that if the fraction of the skilled unemployed category in overall unemployment increases (case of over-education), the instantaneous probability of hiring "p^q" increases ($\frac{\partial p^q}{\partial \beta} \prec 0$), while the probability of leaving unemployment "J^q" decreases ($\frac{\partial J^q}{\partial \beta} \prec 0$), likewise for the unskilled group.

Studies have been advanced close to this direction by examining the presence of an educational investment in the matching process (Blanchard, O. 1999, Moen, E.R. 1999, Charlot, O. 2005).

The comparison of "p^q" and "p^n" means that:

$$p^q - p^n = 1 - e^{\frac{-\beta}{\delta}} - e^{\frac{-\beta}{\delta}}\left(1 - e^{\frac{-(1-\beta)}{\delta}}\right)$$

$$= \left(1 - e^{\frac{-\beta}{\delta}}\right) + e^{\frac{-1}{\delta}}\left(1 - e^{\frac{(1-\beta)}{\delta}}\right) \succ 0$$

So, $p^q \succ p^n$ likewise « $J^q \succ J^n$ ». This shows that the instant probability of hiring and the probability of leaving unemployment are higher among the skilled unemployed than among the unskilled unemployed.[10]

3. Functions of the gains

3.1. Determination of workers' earnings

In the steady-state, there are three situations for an individual in the labor market, either he has a job, or he is in skilled unemployment or unskilled unemployment.

The same hypothesis is made in Blanchard and Diamond (1994), but by analyzing a situation where employers classify workers according to their periods of unemployment. Similar Mortensen and Pissarides (1994), model wage negotiations when a worker can have more than one job offer at the same time. Moen (1997) supposes that firms can simply display the proposed wages, this can reduce the problems related to over-investment.

---

[10] $P(u,v) = v\left(1 - e^{-\frac{u}{v}}\right)$: hiring volume.

The dynamics of the total unemployment rate are as follows: $\dot{u} = \rho(1-u) - P(u,v)$ with ρ : job destruction rate, ρ *(1-u)* : flows of unemployed.

It is assumed that the depreciation of diplomas is done according to the law fish of the parameter "λ", which will result in the transition from skilled unemployment to unskilled unemployment over a period of time.

The amount received by an individual when he is unemployed is: $x = \gamma \bar{s}\,(1-t)$, with $\bar{s}$: the average salary of the economy, γ: the replacement rate, t : the tax rate.

Expectations of earnings of the unemployed, S_u^q *and* S_u^n given:

$$rS_u^q = x + J^q\left(S_e^q - S_x^q\right) + \lambda\left(S_u^n - S_u^q\right) \qquad (8)$$
$$rS_u^n = x + J^n\left(S_e^n - S_u^n\right) \qquad (9)$$

With S_e^q *and* S_e^n are, respectively, the expectations of the discounted income of an employed worker leaving skilled and unskilled unemployment.

It is deduced from equation (***8***) that a skilled unemployed person will be paid based on income "*u*" until he finds a job with a probability «J^q» and with a risk «δ» to be downgraded. The gain on the change of state corresponds to $\left(S_e^q - S_u^q\right)$, and at a loss $\left(S_u^n - S_u^q\right)$ which is due to the depreciation of diplomas.

Similarly, it can be deduced from Equation (***9***) that an unskilled unemployed person is paid based on income «x» until he comes out of unemployment with a probability «J^n», $\left(S_e^n - S_u^n\right)$ is the gain on the change of state.

The skills of an unemployed person on the labor market are represented by the variables to the left of equation (***8***) and (***9***), while the variables on the right represent labor market performance (arbitration relations).

The net salary distributed is:

$$rS_e^i = s^i(1-t) + \rho\left(S_u^i - S_e^i\right) \qquad for\; i = q, n$$

With "s^i": the salary received by category of unemployment,

"ρ": the rate of job destruction,
$\left(S_u^i - S_e^i\right)$: Relative gain after job destruction.

It should be noted that this construction puts that:

- $\left(S_e^q \geq S_u^q\right)$, $\left(S_e^n \geq S_u^n\right)$, $\left(S_e^q \geq S_e^n\right)$, $\left(S_e^n \geq S_u^q\right)$
- That the unemployed are trained at the very moment of hiring.

3.2. Determining the firm income function

The deduction of the earnings function for firms is based on a set of hypotheses. Where it is assumed that firms are risk-averse, offer only one good, and only need one employee.

Similarly, it is assumed that there is a fixed cost to post a vacancy and that employers create positions until the expected value of a filled position equals that fixed cost.

The expected value of a job is given by the probability of fulfilling it by a qualified unemployed person or an unskilled unemployed person through the global matching function, expected earnings generated by labor, and the bargaining power of workers.

The firm will choose its future employees by respecting the rules already mentioned.

The expected earnings from a vacant position "H^v" will be estimated based on the expectation of discounted profits from a job held by a qualified unemployed or an unskilled unemployed "H^q" and "H^n":

$$rH^v = -\sigma + p^q\left(H^q - H^v\right) + p^n\left(H^q - (Z - M) - H^v\right) \qquad \textbf{\textit{(10)}}$$

With σ : cost of the vacancy

Z: training cost.

M: State subsidy, which is defined as follows:

$$M = \frac{\varepsilon^i x}{r / J^i} \qquad i = q, n$$

The subsidy is a proportion "ε" of expected flows that are discounted at an interest rate "r", "x" allocations. For the case where there is no transfer program, we will have "$\varepsilon = 0$".

As the objective of the grants is primarily to reduce training costs, it is assumed that "$Z - M \geq 0$".

The earnings from a long-term vacant job tend to zero "$H^v = 0$", so we can deduce from the equation (***10***):

$$\sigma + p^n (Z - M) = p^q H^q + p^n H^n$$

The expression to the right of the equation explains the gains, which are assumed to be equal in the long term, while the expression to the left explains the costs. The analysis of the decisions of firms in terms of expected gains leads us to the following arbitration equation:

$$rH^i = W - s^i + \rho\left(H^v - H^i\right), \quad i = q, n \qquad \textbf{\textit{(11)}}$$ [11]

With (W - s^i): The profit from production activity obtained by hiring a type of "i" unemployed person.

"ρ": It is a rate which informs us about the possibilities of separation between the company and the employee who belonged to the class of unemployed type "i".

The resolution of (***11***) in "H^i" with "$H^v = 0$" gives:

$$H^i = \frac{W - s^i}{r + \rho} \qquad i = q, n$$

[11] For reasons of simplification it has been assumed that $W^i = W$.

4. Search for balance

The determination of the stationary equilibrium necessarily passes by the analysis of the determinants of wages as well as the flows on the labor market; one also needs what was concluded on the joint behavior of the firms and the unemployed.

4.1.Balance in the labor market

The determination of the status of individuals on the labor market depends on several factors, the behavior of firms and the destruction of jobs being mainly cited.

The probabilities of leaving unemployment on the labor market for skilled and unskilled unemployed are "J^q" and "J^n" respectively, a flow of depreciation from skilled d to unskilled unemployment also recorded with a rate "λ", if it is assumed that firms train their workers, the destruction of jobs increases the number of qualified unemployed at a rate «ρ». Hence the following program:

$$\begin{cases} \rho\left(1-\left(U^q+U^n\right)\right)=U^qJ^q+U^nJ^n & (12) \\ \lambda U^q = J^nU^n & (13) \end{cases}$$

According to (***12***)

$$\Rightarrow \rho\left(1-\left(\beta u+(1-\beta)u\right)\right)=\left(1-e^{-\frac{\beta}{\delta}}\right)\frac{\delta}{\beta}(\beta u)+e^{-\frac{\beta}{\delta}}\left(1-e^{-\frac{(1-\beta)}{\delta}}\right)\frac{\delta}{(1-\beta)}\left(u(1-\beta)\right)$$

$$=\delta\left(1-e^{-\frac{\beta}{\delta}}\right)x+\delta\left(e^{-\frac{\beta}{\delta}}-e^{-\frac{1}{\delta}}\right)u$$

$$=\delta u-\delta u e^{-\frac{1}{\delta}}$$

$$\Rightarrow \rho(1-u)=\left(1-e^{-\frac{1}{\delta}}\right)\delta u \qquad (14)$$

Similarly, according to (***13***):

$$\Rightarrow \lambda\beta u = J^{n}(1-\beta)u$$

$$\Rightarrow \lambda\beta = J^{n}(1-\beta) \qquad (15)$$

We can deduce from the relation (***14***) which expresses the global flows of entry and exit, a strictly decreasing relation between "δ" and "U", explaining an increase of the indicator of tension on the market of work if the overall unemployment rate decreases. Besides, we can subtract from relation (***14***):

$$u = \frac{\rho}{\rho + \delta\left(1 - e^{-\frac{1}{\delta}}\right)}$$

$$\lim_{\delta \to 0} u = \frac{\rho}{\rho + \delta\left(1 - e^{-\frac{1}{\delta}}\right)} = 1$$

This result clearly shows that if the tension on the labor market is low, this leads to a very high unemployment rate.

$$\lim_{\beta \to 0} u = \frac{\rho}{\rho + \delta\left(1 - e^{-\frac{1}{\delta}}\right)} = 0$$

So if jobs develop and adapt to changes, we can arrive at the case where unemployment is located at its lowest level.

And

$$\frac{\partial u}{\partial \delta} = \frac{-\rho\left[\left(1 - e^{-\frac{1}{\delta}}\right) + \delta\left(-\frac{1}{\delta^{2}} e^{-\frac{1}{\delta}}\right)\right]}{\left[\rho + \delta\left(1 - e^{-\frac{1}{\delta}}\right)\right]^{2}} = \frac{-\rho\left[1 - e^{-\frac{1}{\delta}}\left(1 + \frac{1}{\delta}\right)\right]}{\left[\rho + \delta\left(1 - e^{-\frac{1}{\delta}}\right)\right]^{2}} \prec 0$$

Likewise, according to (***15***):

$$\lambda\beta = J^{n}(1-\beta)$$

$$\Rightarrow \lambda\beta = (1-\beta)e^{-\frac{\beta}{\delta}}\left(1-e^{-\frac{(1-\beta)}{\delta}}\right)\frac{\delta}{(1-\beta)} = e^{-\frac{\beta}{\delta}}\left(1-e^{-\frac{(1-\beta)}{\delta}}\right)\delta = \left(e^{-\frac{\beta}{\delta}}-e^{-\frac{1}{\delta}}\right)\delta$$

$$\Rightarrow \lambda\partial\beta = \partial\left[\left(e^{-\frac{\beta}{\delta}}-e^{-\frac{1}{\delta}}\right)\lambda\right] = \delta\partial\left(e^{-\frac{\beta}{\delta}}-e^{-\frac{1}{\delta}}\right)+\left(e^{-\frac{\beta}{\delta}}-e^{-\frac{1}{\delta}}\right)\partial\delta$$

$$\Rightarrow \lambda\frac{\partial\beta}{\partial\delta} = \delta\frac{\partial\left(e^{-\frac{\beta}{\delta}}-e^{-\frac{1}{\delta}}\right)}{\partial\delta}+\left(e^{-\frac{\beta}{\delta}}-e^{-\frac{1}{\delta}}\right) = e^{-\frac{\beta}{\delta}}\left(1+\frac{\beta}{\delta}\right)-e^{-\frac{1}{\delta}}\left(1+\frac{1}{\delta}\right)\succ 0$$

[12]

This result means that the proportion of qualified unemployed in total unemployment increases, if the tension on the labor market increases. We also note:

$\frac{\partial\delta}{\partial u} = \frac{\partial\delta}{\partial\beta}\frac{\partial\beta}{\partial u}\prec 0$, and we have $\frac{\partial\delta}{\partial\beta}\succ 0$ so $\frac{\partial\beta}{\partial u}\prec 0$, explaining a decrease in the proportion of skilled unemployed, if the total unemployment rate increases.

We also deduce that: $u = \dfrac{\rho}{\rho+\lambda\beta\left(\dfrac{1-e^{-\frac{1}{\delta}}}{e^{-\frac{\beta}{\delta}}-e^{-\frac{1}{\delta}}}\right)}$

So, $\lim_{\beta\to 0} u = 1$

If the proportion of qualified unemployed in global unemployment is very small, there will be an increase in overall unemployment. This is explained by the effect of the structure by the qualification of the

---

[12] $(1+u)e^{-u}>(1+w)e^{-w}$ if and only if $u<w$, for our case $\beta<1$.

population on the demand for work. Expressing that the increase in the number of qualified people affects the quality, the type and the number of jobs found.

Saint-paul (1996), Acemoglu (1999) show that the increase in the supply of skilled labor leads to a new segmentation of the labor market, the effect on overall unemployment depends on the size of each category.

4.2. Calculation of wages

The process of wage negotiations between firms and the unemployed necessarily involves maximizing the weighted product of net earnings. The gains made by a firm as a result of hiring a skilled unemployed worker or another unskilled worker are respectively: $(H^q - H^v), (H^n - (Z - M) - H^v)$.

Similarly, the earnings of the skilled and unskilled unemployed are respectively: $(S_e^q - S_u^q), (S_e^n - S_u^n)$, we, therefore, determine the optimal wages from these two programs:

$$\underset{s^q}{Max} \left(S_e^q - S_u^q\right)^{\alpha} \left(H^q - H^v\right)^{1-\alpha} \qquad (16)$$

$$\underset{s^n}{Max} \left(S_e^n - S_u^n\right)^{\alpha} \left(H^n - (Z - M) - H^v\right)^{1-\alpha} \qquad (17)$$

With $\alpha \in [0 , 1]$, which explains the bargaining power of the unemployed.

The first order maximization condition on the program (***16***) gives:

$$\alpha\left(\frac{1-t}{r+\rho}\right)\left[s^q\left(\frac{1-t}{r+\rho}\right) - S_u^q\left(\frac{r}{r+\rho}\right)\right]^{\alpha-1}\left(\frac{Y-s^q}{r+\rho}\right)^{1-\alpha}$$

$$-(1-\alpha)\left(\frac{1}{r+\rho}\right)\left(\frac{W-s^q}{r+\rho}\right)^{-\alpha}\left[s^q\left(\frac{1-t}{r+\sigma}\right) - S_u^q\left(\frac{r}{r+\rho}\right)\right]^{\alpha} = 0$$

$$\Rightarrow \alpha\left(\frac{1-t}{r+\rho}\right)\left[s^q\left(\frac{1-t}{r+\rho}\right)-S_u^q\left(\frac{r}{r+\rho}\right)\right]^{\alpha-1}\left(\frac{W-s^q}{r+\rho}\right)^{1-\alpha}$$

$$=(1-\alpha)\left(\frac{1}{r+\rho}\right)\left(\frac{W-s^q}{r+\rho}\right)^{-\alpha}\left[s^q\left(\frac{1-t}{r+\rho}\right)-S_u^q\left(\frac{r}{r+\rho}\right)\right]^{\alpha}$$

$$\Rightarrow \alpha(1-t)\left(\frac{W-s^q}{r+\rho}\right)=(1-\alpha)\left[s^q\left(\frac{1-t}{r+\rho}\right)-S_u^q\left(\frac{r}{r+\rho}\right)\right]$$

$$\Rightarrow \alpha(1-t)\left(W-s^q\right)-(1-t)(1-\alpha)s^q=-(1-\alpha)rS_u^q$$

$$\Rightarrow \alpha(1-t)s^q+s^q(1-t)(1-\alpha)=\alpha(1-t)W+(1-\alpha)rS_u^q$$

$$\Rightarrow (1-t)s^q=\alpha(1-t)W+(1-\alpha)rS_u^q \qquad (18)$$

We deduce from the first order maximization condition of (***17)***:

$$\alpha\left(\frac{1-t}{r+\rho}\right)\left[s^n\left(\frac{1-t}{r+\rho}\right)-S_u^n\left(\frac{r}{r+\rho}\right)\right]^{\alpha-1}\left(\frac{W-s^n}{r+\rho}-(Z-M)\right)^{1-\alpha}$$

$$=(1-\alpha)\left(\frac{1}{r+\rho}\right)\left(\frac{W-s^n}{r+\rho}-(Z-M)\right)^{-\alpha}\left[s^n\left(\frac{1-t}{r+\rho}\right)-S_u^q\left(\frac{r}{r+\rho}\right)\right]^{\alpha}$$

$$\Rightarrow \alpha(1-t)\left(\frac{W-s^n}{r+\rho}-(Z-M)\right)=(1-\alpha)\left[s^n\left(\frac{1-t}{r+\rho}\right)-S_u^n\left(\frac{r}{r+\rho}\right)\right]$$

$$\Rightarrow (1-t)s^n=\alpha(1-t)W-(Z-M)(r+\rho)(1-t)\alpha+(1-\alpha)rS_u^n \qquad (19)$$

According to (***18***) and (***19***), the negotiated wage is determined according to the production (*W*), the bargaining power (α) and ($S_u^i, i=q,n$), the latter expresses the limits of the arbitration between skilled and unskilled unemployed. We note that (S_u^i) subsidy policies increase negotiated wages.

We also note that for unskilled workers, the negotiated salary is weakened by training costs

(Z-M). And since $S_u^q \succ S_u^n$, according to the previous section, we can write that we have always $s^q \succ s^n$.

4.3.Stationary equilibrium

For search the stationary equilibrium, it suffices to extract the State's budgetary constraint:

- Subsidies distributed $(M\, p^n\, k\, v)$,
-
- Unemployment allowances paid $(x\, u\, k)$,
- Taxes levied on wages $(t\,(T^q s^q + T^n s^n)\, k)$, with $T^q = \frac{J^q \alpha\beta u}{\rho}$ and $T^n = \frac{J^n (1-\beta) u}{\rho}$ checking
  $T^q + T^n = 0$, the state financing constraint is therefore:

$$t\left(T^q s^q + T^n s^n\right)k = x\,u\,k + \delta\,u\,p^n\,M\,k$$
$$\Rightarrow t\left(T^q s^q + T^n s^n\right) = x\,u + \delta\,u\,p^n\,M$$

The introduction of the budget constraint in the model assumes that the State fixes the replacement rate and that the replacement rate is endogenous and varies according to the fluctuation of the equilibrium on the labor market.[13]

Either the average salary of the economy: $\bar{s} = \frac{T^q s^q + T^n s^n}{1-u}$.

From this budget balance, we can look for the replacement rate *(γ)* who checks:

[13] In the opposite case where the contribution rate is variable while the replacement rate is fixed, the tax burden will be heavier, especially for a high unemployment rate.

$$t\,\bar{s}\,(1-u) = \bar{s}\gamma\,(1-t)u + \delta\, u\, p^{n}\,\varepsilon\,\frac{\bar{s}\gamma\,(1-t)}{r/J^{n}}$$

$$\Rightarrow \gamma = \frac{(1-u)t}{(1-t)\left(u + \dfrac{\delta\, u\, p^{n}\,\varepsilon}{r/J^{n}}\right)}$$

$$\Rightarrow \gamma = \frac{(1-u)t}{(1-t)u\left(1 + \dfrac{\delta\, p^{n}\,\varepsilon}{r/J^{n}}\right)}$$

The stationary equilibrium constructed takes account of the replacement rate (γ), and satisfies the following conditions:

- No barrier to entry, relation ***(15)***,
- The joint equilibrium of workers and firms balance of flows, relations ***(16)***, ***(17)***,
- The first-order conditions of wage maximization, relations (***18***), (***19***),
- All of the following endogenous variables: β, δ, u, s^q, s^n.

5. Supputation

The objective of this paragraph is the analysis of the impact of the benefit transfer policy (ε), on skilled and unskilled unemployment, on wages (s^q, s^n), on hiring probabilities (P^q, P^n), on expected utilities (S^q, S^n), and unemployment exit rates (J^q, J^n).

The model is applied using data relating to the Tunisian economy. The case of no transfer of benefits represents the initial reference case ($\varepsilon = 0$), with a total unemployment rate of 15.4%, of which 29.2% are skilled unemployed, the discount rate (r = 6.75%), the replacement rate (γ = 0.554), the transition rate from skilled unemployment to unskilled unemployment (λ = 1/5 = 0.20), bargaining power (α = 0.4), then the productivity is normalized to the unit ($W = 1$).

From the initial state (ε = 0), we can deduce from expression (γ) the contribution rate "*t*" which guarantees the budgetary equilibrium of the State, ie t = 9.88%, we also calculate (δ = 0.163).

From equation (*16*) we can calculate the job destruction rate (ρ = 6.3%), the training cost is deducted for the value of (a = 1) which gives (Z-M = 0), therefore (Z = 0.3765).

We also deduce:

p^q = 0.8332, p^n = 0.1647, J^q = 0.4654, J^n = 0.0379, s^q = 0.9051, s^n = 0.9051 [14]

For a vacant job, the postage cost (respecting the conditions of free entry) is *(σ = 0.705).*

$$S_u^i \approx \frac{(1-t)}{r}s^i - \frac{\rho}{r}s^i \quad with\ i = q,n$$

[14] $$s^q = \frac{\alpha(1-t)}{(1-t)-(1-\alpha)(1-t)+\rho(1-\alpha)}W$$

$$t = \frac{\gamma u\left(r+\delta\varepsilon p^n J^n\right)}{\gamma u\left(r+\delta\varepsilon p^n J^n\right)+r(1-u)}$$

$$s^n\left[(1-t)-(1-\alpha)(1-t-\rho)-\frac{\varepsilon\left(J^n\right)^2 u\alpha\gamma}{\rho r(1-u)}(r+\rho)(1-t)^2(1-\beta)\right]=$$

$$\alpha(1-t)W - Z(r+\rho)(1-t)\alpha - \varepsilon J^n J^q \beta u\gamma\alpha s^q \frac{(r+\rho)(1-t)^2}{r\rho(1-u)}$$

Table 1: The effect of the benefit transfer policy on the equilibrium in the standard model (fixed contribution rate model).

ε	0	0.3	0.5	0.7	1
t	0,0916	0,0937	0,0952	0,0966	0,0988
s^q	0,90577	0,9055	0,9054	0,9052	0,9051
s^n	0,818	0,8744	0,8832	0,8919	0,9051
Z-M	0.37654	0.2635	0.1882	0.113	0
S^q	11,3442	11,31	11,291	11,27	11,24
S^n	10,245	10,92	11,0144	11,104	11,24

Figure 1: Skilled wages

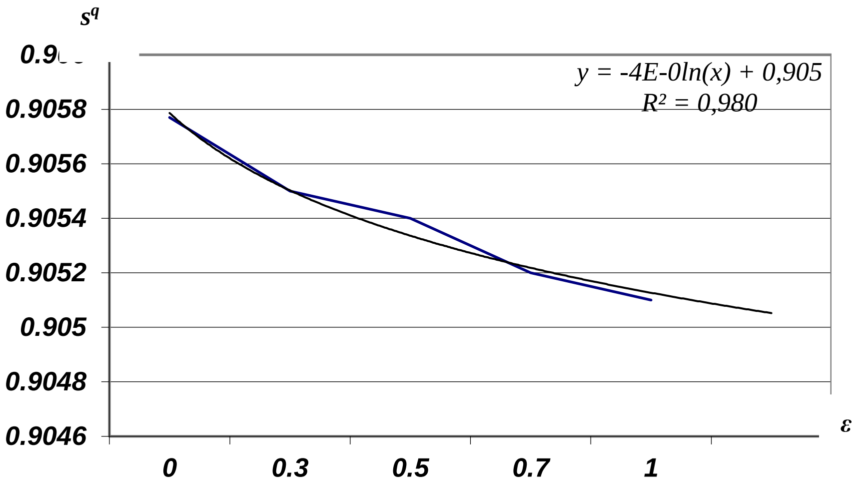
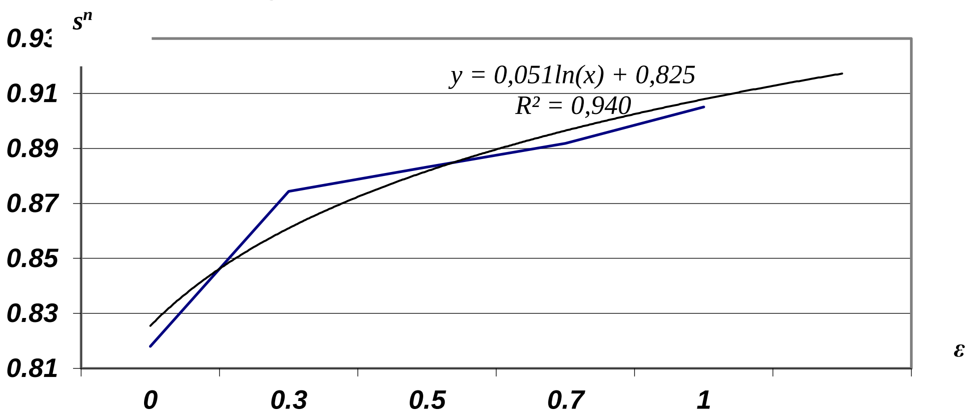


Figure 2: Unqualified wages



From these two curves, it appears that an increase of *(ε)* led on one side to a decrease of *(s^q)*, and on the other hand, to a double effect on *(s^n)*. In the first phase, there is a rapid increase and in the second phase an increase, but slower than the first phase. Thus, such a reallocation policy favors the unskilled unemployed to the detriment of the skilled unemployed, mainly for *(*$0 < \varepsilon < 0.3$*)* where this policy is fully effective in this area. It should be noted that *(s^q)* is always superior to *(s^n)*, whatever *(*$\varepsilon<1$*)*.

The present situation has the effect of reducing the wage differences between skilled and unskilled workers, and therefore increasing collective well-being. But on the other hand, this situation can discourage the most skilled candidates, since the excess salary which makes them stand out compared to the unskilled is no longer attractive, which also weakens their negotiating powers.

The analysis of the expected utility leads to the same results:

Figure 3: Expected Benefits of Skilled

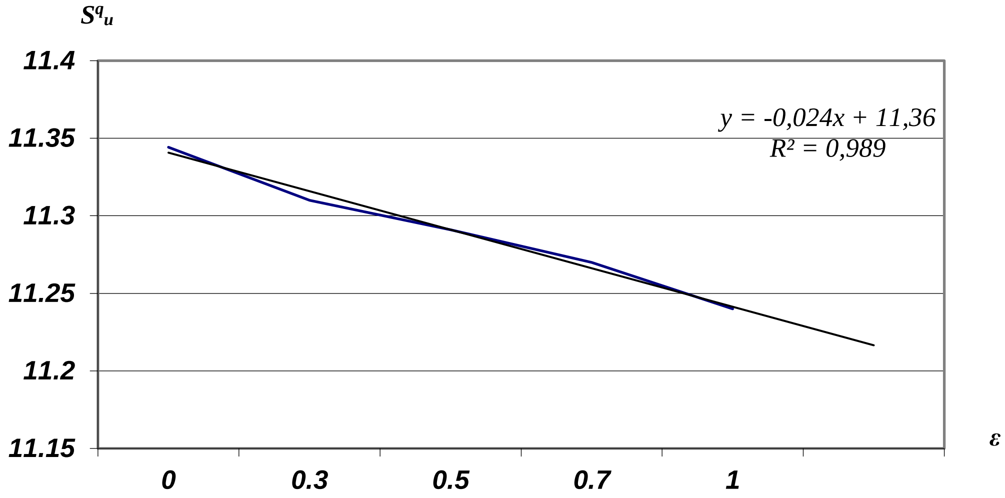


Figure 4: Expected benefits of unskilled

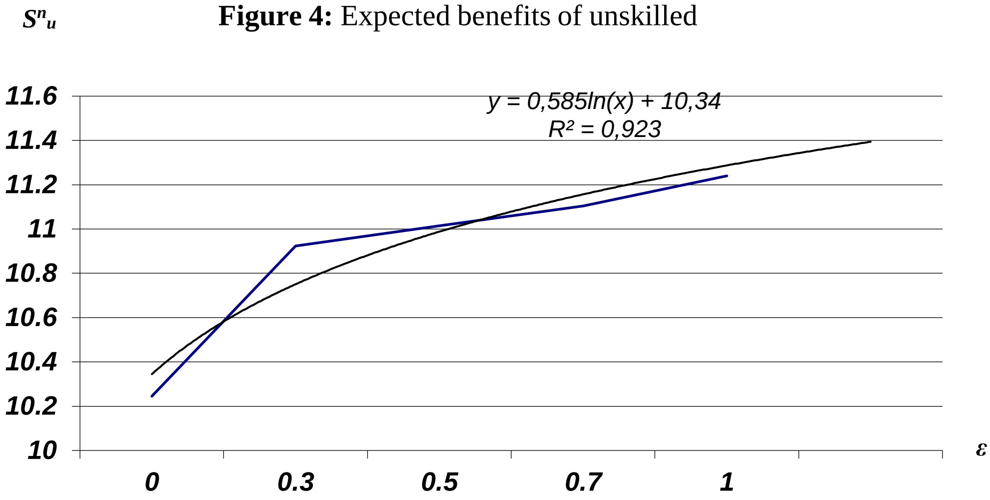
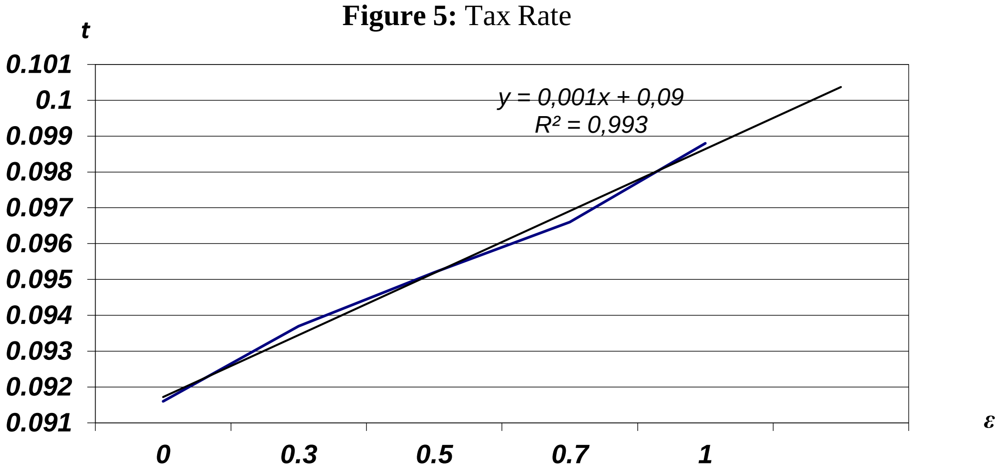


Figure 5: Tax Rate



For firms, the increase in subsidies reduces the recruitment costs, which encourages firms to recruit more and there will be an increase in vacancies and subsequently a decrease in unemployment, especially before reaching the point of inflection (ε = 0.3).

The study of the impact of the subsidies on the probability of hiring and the exit from unemployment can be deduced from the effect of the subsidies on the tension of the labor market. Where, the increase in this tension leads to a decrease in hiring rates, and an increase in unemployment exit rates.

$$\frac{\partial p^q}{\partial \delta} \prec 0, \frac{\partial p^n}{\partial \delta} \prec 0 \quad \text{and} \quad \frac{\partial J^q}{\partial \delta} \succ 0, \frac{\partial J^n}{\partial \delta} \succ 0 .$$

6. Conclusion

The matching theory has come a long way since the early 1980s (A. Pissarides, 2010), where its role was to address the mismatch between job seekers and recruiting employers, and the degree of matching. Matching efficiency will be judged by the unemployment rate. In any case, the presence of a high unemployment rate is not necessarily a sign of a decline in efficiency, so that vacancies are higher for a given category of unemployment.

This study attempts to open up a new line of thinking which relates to matching theory, the aim of which is to treat the position of the unemployed on the labor market, which they are classified as skilled and unskilled.

The analysis shows that an increase in the tension on the labor market generates a reduction in the probability of leaving unemployment and an increase in the probability of hiring.

We have also defined the earnings functions of firms and the unemployed (R. Shimer, 2007). It is shown that the equilibrium of the model can take different forms, depending on the new specification. At Equilibrium, it is shown that if the tension on the labor market increases the proportion of skilled unemployed in total unemployment increases. And the salary will be determined on the basis of production, the bargaining power of the unemployed, and the reserve salary.

It is also important to emphasize that a separation shock generates an increase in unemployment and vacancies. The standard model predicts that a hiring subsidy would slightly reduce overall unemployment (B. Ismail, 2013).

Adaptation of the model with the statistics corresponding to the Tunisian labor market shows that the exercise of a reallocation policy weakens the bargaining power of the unemployed and decrease their wages while increasing the wages of the unskilled, in particular for (ε <0.3)

Acknowledgement

I would like to thank the anonymous reporters of the journal for their suggestions and remarks to carry out this work, and the author remains solely responsible for any possible errors.

References

Acemoglu, D. (1999). Changes in Unemployment and Wage Inequality: An Alternative Theory and Some Evidence, *American Economic Review*, 89(5), pp. 1259-1278.

Albrecht, J.;Vroman, S. (2002). A matching model with endogenous skill requirements, *International Economic Review*, 43, pp. 283–305.

Ashenfelter, O.; Kruger, A. (1994). Estimates of the economic return to schooling from a new sample of twins, *American Economic Review*, 84, pp. 1157-1173.

B*arnichon*, R.; A*ndrrew*, F. (2015). Labor Market Heterogeneity and the Aggregate Matching Function: Dataset, *American Economic Journal: Macroeconomics*, 7(4), pp. 222–249.

Blanchard, O.; Diamond, P.; Hall, R.; Murphy K. (1990). The Cyclical Behavior of the Gross Flows of U.S. Workers, *Brookings Papers on Economic Activity*, 21(2), pp. 85-156.

Blanchard, O.; Diamond, P. (1994). Ranking, Unemployment Duration, and Wages, *Review of Economic Studies*, 6, pp. 417-434

Blanchard, O.; Diamond, P. (1991). The Aggregate Matching Function, *NBER Working Papers*, 3175, National Bureau of Economic Research, Inc.

Blanchard, O.; Quah, D. (1989). The Dynamic Effects of Aggregate Demand and Supply Disturbances, *American Economic Review*, 79(4), pp. 655-673.

Burdett, K.; Mortensen, D. (1998). Wage Differentials, Employer Size, and Unemployment, *International Economic Review*, 39(2), pp. 257-273.

Davis, S.; Haltiwanger, J. (1999). On the Driving Forces Behind Cyclical Movements in Employment and Job Reallocation, *American Economic Review*, 89(5), pp. 1234-1258.

Davidson, C.; Matusz, S.; Shevchenk, O. (2007). Globalization and firm level ad-justment with imperfect labor markets, Working paper, Michigan State University, Lansing.

Diamond, A. (1981). Mobility Costs, Frictional Unemployment, and Efficiency, *Journal of Political Economy*, 89(4), pp. 798-812.

Gautier, P.;Van den Berg, G.; VAN OURS, J.; G. RIDDER, G. (2002). Worker turnover at the firm level and the crowding out of lower educated workers, European Economic Review, 46, pp. 523–538.

Hagedorn, (M.; Manovskii, I. (2008). The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited, *American Economic Review*, 98(4), pp. 1692-1706.

Hosios, A. (1990). On the Efficiency of Matching and Related Models of Search and Unemployment, *Review of Economic Studies*, 57(2), pp. 279-298.

Moen, E. (1997). Competitive Search Equilibrium, *Journal of Political Economy*, Vol. 105(2), pp. 385-411.

Moen, E. (1999). Education, Ranking, and Competition for Jobs, *Journal of Labor Economics*, 17(4), pp. 694-723.

Montgomery, J. (1991). Equilibrium Wage Dispersion and Interindustry Wage Differentials, *Quarterly Journal of Economics*, 106(1), pp. 163-179.

Mortensen, D. (1977). Unemployment insurance and job search decisions, *Industrial and Labor Relations Review*, 30(4), pp. 505-517.

Mortensen, D. (1982). Property Rights and Efficiency in Mating, Racing, and Related Games, *American Economic Review*, 72(5), pp. 968-979.

Mortensen, D.; Nagypal, E. (2007). More on Unemployment and Vacancy Fluctuations, *Review of Economic Dynamics*, 10(3), pp. 327-347.

Mortensen, D.; Pissarides, C. (1999). Job Creation and Job Destruction in the Theory of Unemployment, *Review of Economic Studies*, 61(3), pp. 397-415.

Mortensen, D.; Pissarides, C. (1999). New developments in models of search in the labor market, *Handbook of Labor Economics*, 3(39), pp. 2567-2627.

Mortensen, D.; Pissarides, C. (1999). Job reallocation, employment fluctuations and unemployment, *Handbook of Macroeconomics*, 1(18), pp. 1171-1228.

Mortensen, D. (1987). Job search and labor market analysis, *Handbook of Labor Economics*, 2(15), pp. 849-919.

Petrongolo, B.; Pissarides, C. (2001). Looking into the Black Box : A Survey of the Matching Function, *Journal of Economic Literature*, 39(2), pp. 390-431.

Pissarides, C. (1979). Job Matchings with State Employment Agencies and Random Search, *Economic Journal*, 89(356), pp. 818-833.

Pissarides, C. (1985). Short-run Equilibrium Dynamics of Unemployment Vacancies, and Real Wages, *American Economic Review*, 75(4), pp. 676-690.

pissarides, C. (2000). Equilibrium Unemployment Theory, 2nd Edition, MIT Press Books, The MIT Press, edition 1, Vol. 1, number 0262161877.

Pissarides, C. (2001). Employment Protection, *Labor Economics*, 8, pp. 131-159.

Pissarides, C. (2009). The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer ?, *Econometrica*, 77(5), pp. 1339-1369.

Postel-Vinay, F.; Robin, J. (2002). Equilibrium Wage Dispersion with Worker and Employer Heterogeneity, *Econometrica*.

Sahin, A.; Joseph, S.; Giorgio, T.;Giovanni, L. (2014). Mismatch Unemployment, *American Economic Review*, 104 (11), pp. 3529–64.
Shimer, R. (2005). The Cyclical Behavior of Equilibrium Unemployment and Vacancies, *American Economic Review*, 95(1), pp. 25-49.

Shimer, R. (2007). Reassessing the Ins and Outs of Unemployment, NBER Working Papers 13421, National Bureau of Economic Research, Inc.

Shimer, R. **(**2012). Reassessing the Ins and Outs of Unemployment, *Review of Economic Dynamics*, 15(2), pp. 127–48.

Yashiv, E. (2000). The Determinants of Equilibrium Unemployment , *American Economic Review*, 90(5), pp. 1297-1322.

Appendix

$$\frac{\partial p^q}{\partial \beta} = \frac{1}{\delta} e^{-\frac{\beta}{\delta}} \succ 0, \quad \frac{\partial p^n}{\partial \beta} = -\frac{1}{\delta} e^{-\frac{\beta}{\delta}} \prec 0, \quad \frac{\partial p^q}{\partial \delta} = -\frac{\beta}{\delta^2} e^{-\frac{\beta}{\delta}} \prec 0$$

$$\frac{\partial p^n}{\partial \delta} = \frac{1}{\delta^2} e^{-\frac{\beta}{\delta}} \left(\alpha - e^{-\frac{(1-\beta)}{\delta}} \right) = f(\beta)$$

$$\frac{\partial f(\beta)}{\partial \beta} = \frac{1}{\delta^2} e^{-\frac{\beta}{\delta}} \left(1 - \frac{\beta}{\delta} \right)$$

"*f*" is increasing (resp decreasing) so:

$$\frac{\partial p^n}{\partial \delta} \succ 0, \ (resp \frac{\partial p^n}{\partial \delta} \prec 0) iff \ \beta \prec \delta \, (resp \, \beta \succ \delta)$$
$$\Rightarrow u^q \succ v \, (resp \, u^q \prec v).$$