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A method for preparing multi-regional input-output tables despite data limitation: FLQ-Gravity

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ABSTRACT

The preparation of multi-regional input-output tables is of great importance because of its multiple applications. However, due to the lack of data in many developing countries, including Iran, its preparation faces serious challenges. To deal with these challenges, this paper introduces the FLQ-Gravity combination method. The contribution of this research is the use of the basic model of gravity. When there is no interregional trade data, this model can partially estimate interregional trade based on the three factors of production, demand and distance. For this purpose, first, a single region input-output table is prepared by FLQ method. Then, trade between regions is estimated using the gravity method .In the experimental section, the multi-regional input-output table (Tehran, oil-rich regions, and the rest of economies for Iran is prepared using the above method. Then, modeling and calculation are performed on multipliers coefficients and spillover & feedback effects.

ملخص

يكتسي إعداد جداول متعددة المدخلات والمخرجات الإقليمية أهمية كبيرة بسبب تطبيقاتها المتعددة. ولكن بسبب نقص البيانات في عدد من البلدان النامية، بما فيها إيران، فإن إعدادها يواجه تحديات جدية. ولمواجهة هذه الأخيرة، تقدم هذه الورقة أسلوب الجمع بين حاصل الموقع لفليغ والجاذبية (FLQ-Gravity). وتتمثل مساهمة هذا البحث في استخدام النموذج الأساسي للجاذبية. فعندما لا توجد بيانات تجارية أقاليمية، يمكن لهذا النموذج أن يقدر جزئيا التجارة الأقاليمية استنادا إلى

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العوامل الثلاث المتمثلة في الإنتاج والطلب والمسافة.ولهذا الغرض، يتم أولا إعداد جدول إقليمي للمدخلات والمخرجات حسب أسلوب حاصل الموقع لفليغ. ثم يتم تقدير التجارة بين المناطق باستخدام طريقة الجاذبية. ففي القسم التجريبي، يتم إعداد جدول المدخلات والمخرجات بين المناطق المتعددة (طهران، المناطق الغنية بالنفط، وبقية الاقتصادات بالنسبة لإيران) باستخدام الطريقة المذكورة أعلاه. ثم، يتم النمذجة والحساب على معاملات المضاعفات والتأثيرات غير المباشرة والمرتدة.

ABSTRAITE

La préparation de tableaux d'entrées-sorties multirégionaux revêt une grande importance en raison de ses multiples applications. Cependant, en raison du manque de données dans de nombreux pays en développement, dont l'Iran, sa préparation est confrontée à de sérieux défis. Pour faire face à ces défis, cet article introduit la méthode de combinaison FLQ-Gravity. La contribution de cette recherche est l'utilisation du modèle de base de la gravité. Lorsqu'il n'existe pas de données sur le commerce interrégional, ce modèle permet d'estimer partiellement le commerce interrégional sur la base des trois facteurs que sont la production, la demande et la distance. À cette fin, un tableau d'entrées-sorties pour une seule région est d'abord préparé par la méthode FLQ. Ensuite, le commerce entre les régions est estimé en utilisant la méthode de gravité. Dans la section expérimentale, le tableau des entrées-sorties multirégional (Téhéran, régions riches en pétrole et le reste des économies pour l'Iran) est préparé en utilisant la méthode ci-dessus. Ensuite, la modélisation et le calcul sont effectués sur les coefficients des multiplicateurs et les effets d'entraînement et de rétroaction.

Keywords: Multi-regional input-output table; Location quotient method; Gravity model; Spillover effects; Feedback effects

JEL Classification: C67, R15, R12

1. Introduction

The input-output model is one of the economic policy-making models that can make policies at the national and regional levels. Regional tables are prepared as single-regional, inter-regional, and multi-regional (three-regional and more) tables. The calculation of multi-regional input-output coefficients is of great importance for determining the multipliers coefficients and spillover and feedback effects. The main challenge in using this model is its preparation despite the lack of data at the regional level. In this regard, researchers have prepared a multi-regional table based on data available in their country (e.g., Yamada (2015) in Japan and Boero et al. (2018) in Germany). Regarding the lack of data at the regional level in most developing countries (including Iran), researchers have to prepare multi-regional input-output tables, taking into account the above limitations. In this regard, Isard (1951) states: “Multiregional Input-Output tables usually need to be estimated due to the unavailability of subnational level data”.

In addition, the type of input-output table preparation varies according to its application. The location quotient is a suitable method when the goal is to analyze the production, employment, and structural changes of one or more regions (Flegg and Tohmo, 2018). Flegg Location Quotient (FLQ) is the most appropriate method of location quotient. This method simultaneously accounts for the size of the selling sectors, the size of the purchasing sectors, and the size of the region (Kowalewski, 2015). The focus of FLQ is on the output generated within a specific region. So, it should be only being used in conjunction with national IO tables where the inter-industry transactions exclude imports.

Calculating inter-regional trade is another important issue in the preparation of multi-regional input-output tables. Gravity is one of the models that are widely used today in calculating the trade (export and import) coefficients of multi-regional input-output. In recent years, the gravity model has become a workhorse for quantitative studies of international trade and investment policy. This model is based on Newton’s theory of Gravity and states that more Gravity means more trade between regions. The amount of inter-regional trade depends on the level of production, demand, and the distance between them (Miller and Blair, 2009). The Gravity model was first introduced into the regional input-output literature by Leontief and Strout (1963). So far, various modifications have been made to the Gravity model according to the available data of the countries, some of which can be found in the studies conducted by Boero et al. (2018) in the United States, Krebs (2018) in Germany and Yamada (2015) in Japan.

Given the time series data of inter-regional and intra-regional trade in these countries, the model of Gravity can be expanded. In other words, given the data of interregional trade and production of economic sectors in United States, Germany and Japan, the authors have been able to estimate the elasticity (coefficient) of each of these factors and thus provide a modified type of gravity model[1]. However, this is not the case in many developing countries such as Iran. In contrast to international trade, it is still difficult to find regional trade statistics within Iran. Given that the gravity model continues to be very popular and suitable, it is used in this research. An important contribution of the present paper is the preparation of a multi-regional input-output table, despite a lack of inter-regional and intra-regional trade statistics. The focus of the present study is on the innovative approach proposed by Tarahomi and Bazzazan (2020) in Iran. In this approach, two methods of estimation (i.e., the FLQ and the Gravity) are combined to estimate the intraregional and interregional transactions, respectively. A key facet of this approach is the use of the FLQ in estimating intraregional trade and a Gravity model in estimating interregional trade. Hence, the regionalization process comprises two steps: the estimation of intra-regional flows and interregional trade. Intra-regional flows are calculated by the FLQ method and interregional trade flows by the gravity method. In general, this paper provides empirical evidence for the applicability of the FLQ-Gravity method to three regions of Iran. The first objective of this paper is to introduction and use of the combined method of FLQ and gravity methods in order to construct a multi-regional input-output table. The second goal is to model a multi-regional input-output table to determine the effects of spillover and feedback in the three regions. Spillover and feedback effects are important interregional interactions. The spillover effects can be traced back to the theories of economic equilibrium and non-equilibrium development. In the 1950s, economists focused on the growth pole and put forward the spillover effects of the center to the periphery. With the development of new economic growth theory, Romer (1986) and Robert (1988) proposed that the spillover effects are the exogenous economic essence. The economic spillover effects refer to the unidirectional influence of the economic development of one region on the economic development of other regions. The feedback effects are the influence of other regions on the development of the region (Li et al, 2018). Multi-regional input-output models are one of the most widely used approaches to analyze the economic interdependence between different regions. The main question of this research is how in the lack of trade data between regions, gravity method can help to build a multi-regional input-output table. Unfortunately, the previous studies in Iran suffer from several limitations.

[1] In the Gravity Model section, this point will be discussed more.

Most of them have used single-region and two-region input-output tables, and multi-regional analysis has not been performed in Iran. Meanwhile, Iran has 30 provinces that have different economic structures and play different roles in economic growth. The remainder of this paper is organized as follows. In Section 2, the theoretical background of the FLQ and Gravity is briefly recalled. Then, the combination of FLQ and Gravity methods is explained. Modeling multipliers coefficients and spillover & feedback effects in a multi-regional input-output table are described in Section 3. In Section 4, a multi-regional table is prepared and multipliers coefficients and spillover and feedback effects are calculated. Finally, in Section 5, the results and suggestions are presented.

2. Literature Review

As mentioned in the introduction, the method used is a combination of FLQ and Gravity methods. First, the FLQ method is explained and then the Gravity method is discussed. In this research, the following data are used to construct a multi-region input-output table:
- Input-output table of Iran. The latest table published in Iran is related to the year 2011.
- Statistics of intermediate consumption, value added and output of 18 economic sectors are studied. The above data has been published by the Statistics Center of Iran.
- Statistics related to the ground distance of the centers of the studied provinces (for use in the gravity model)[1].

2.1. FLQ method

Where regional IO tables are unavailable, survey-based methods are surely the most reliable way of producing them, yet this would go well beyond the time and funding limits of most research projects. A simple and cheap alternative is to employ LQ-based techniques. In applying LQ, the national transactions matrix is first transformed into an input coefficient matrix. LQ-based techniques assume that the intraregional input coefficients in any region (b_{ijr}), depend on the corresponding national input coefficients (a_{ijr}), as follows:

[1] It should be noted that in all research in the field of input-output, the mentioned data has been used.

$b_{ijr} = LQ_{ijr} * a_{ijr}$ (1)

Here b_{ijr} can be interpreted as the amount of input from sectors i in region r that is required by sector j in that region to produce one unit of gross output.

LQ_{ijr} represents an LQ-based formula, such as the FLQ, the simple LQ (SLQ) or cross industry LQ (CILQ). Formula (1) entails that, where $LQ_{ijr} \geq 1$ the intraregional coefficients are equated to the corresponding national coefficients. Otherwise, the b_{ijr} are adjusted downwards to take account of inputs imported from other regions (Kowalewski, 2015).
The SLQ is defined here as:

$$SLQ_i^r = \frac{Q_i^r/Q^r}{Q_i/Q} \quad (2)$$

Where Q_i^r denotes the output of regional sector i, while Q_i is the corresponding national figure. Q^r and Q represent the overall output of region r and the nation. The CILQ is defined as:

$$CILQ_{ij}^r = \frac{SLQ_i^r}{SLQ_j^r} \quad (3)$$

Where i and j refer to the supplying and purchasing sectors, respectively. As is customary, where i=j, we have set $CILQ_{ij}^r = SLQ_i^r$.
However, Empirical research has demonstrated that the SLQ, CILQ and related formulae tend to understate a region's imports from other regions and hence tend to overstate multiplier effects (Flegg & Tohmo,2013).The FLQ aims to overcome this shortcoming by taking regional size into account. The FLQ is defined here as:

$$FLQ_{ij}^r = CILQ_{ij}^r * \lambda^r \quad (4)$$

Where

$$\lambda^r = [log_2(1 + Q^r/Q)]^\delta \quad (5)$$

Q^r/Q Measures regional size in this equation, while the parameter δ controls its convexity. It is assumed that $0 \leq \delta < 1$. As δ rises, so too does the allowance for imports (from other regions or from abroad). δ=0 represent a special case where $FLQ_{ij}^{r} = CILQ_{ij}^{r}$.

The value of δ is calculated by various methods and using data available in each country. For example, Kowalewski (2015) has used an econometric equation to estimate its value for each segment (Malte et al, 2018). The FLQ method is used to prepare single-regional input-output tables. Besides, to prepare inter-regional trade, the method of Gravity is required, which is discussed below.

2.2. Gravity Model and its compliance with Iranian economic data

The model of Gravity introduced by Leontief & Strout (1963) is in the form of Eq. (6) (Miller & Blair, 2009).

$$T_i^{rs} = \frac{X_i^r * X_i^s}{X_i} * Q_i^{rs} \quad , \quad Q_i^{rs} = \frac{k_i^{rs}}{(d^{rs})^{e_i}} \tag{6}$$

Where T_i^{rs} is the intra-regional trade exchange of sector i between the two regions r and s. X_i^r is the total supply of sector i in region r. X_i^s is the total demand of sector i in region s. X_i is the total production in sector i in two regions. Q_i^{rs} is a component that consists of two elements k_i^{rs} and d^{rs}. k_i^{rs} is a constant coefficient element. d^{rs} is the distance between two regions r and s. e_i is also a distance power (elasticity). Using this equation, due to data limitations and experimental studies, faces the following constraints[1]:

[1] In fact, it can be said that in this section, the simplification of the gravity model has been done according to the available data in the Iranian economy. This section covers the gap caused by the lack of trade data between regions. Obviously, it can be used for countries that have such a situation in terms of statistics.

a) In this equation, the amount of supply elasticity and demand elasticity is considered equal to 1. However, according to recent input-output research, this issue has been reviewed and researchers have defined each of the variables of Eq. (6) with separate power (elasticity) for each sector (Li et al. 2018; Krebs et al. 2018). Considering a separate elasticity for each region is because the impact of demand and supply varies according to factors such as consumption and taste (on the demand side) and technology and production structure (on the supply side).

b) In Eq. (6), determining the value of k_i^{rs} is a challenging issue. Experimental studies show that its value is considered constant between different sectors of a region and between different regions.

c) Determining the value of distance elasticity (the value of e_i) is another challenge of using the Gravity model. According to most experimental studies, its value is considered as unity (Anderson, 2011 and 2016; Chaney, 2011).

d) It is difficult to determine the distance when several provinces are studied in the form of a region. In this research, according to experimental studies, to determine the distance, the arithmetic average distance of provincial centers is used.

Despite the above constraints, using this method has the following advantages:

1) It is possible to create multi-regional input-output tables using this method.

2) Since the Gravity model calculates the trade relationship between the two regions well, it can also be used to prepare multi-region input-output tables from which the national economy is not obtained.

The Gravity model has been used in many new studies, which are referred to below.

Gabela (2020) in an article examining the method of gravity in japan. Results show high overall accuracy levels for this approach. Due to the existence of intraregional and interregional trade statistics in Japan, the author has been able to estimate the coefficient (elasticity) of each of the elements of Equation (6) in this article.

Kerbs (2018) has used Equation 6 of this research to prepare a multi-regional input-output table in Germany. Elasticity of production and demand in the model of gravity is considered equivalent to a unit, but the elasticity of distance is estimated by econometric method. The elasticity of the distance between economic sectors is calculated from 1.07 to 2.87.

Mi et al. (2018) prepared the 30 regional input-output table of China for 2012 using the gravity method. They have used Equation 6 of this research and the production, demand and distance elasticity for each sector have been estimated.

Safr & Sixta (2018) has examined trade between different regions of the Czech Republic using various models including gravity. They have used Equation 6 of this research and have estimated production, demand and distance elasticity using econometric model.

As mentioned, due to the lack of time series trade data between regions in Iran, econometric estimation is not possible. Therefore, Equation 6 is used to estimate trade between regions. In addition, in the Japan, Germany, China and Czech Republic countries (Which were mentioned in the above studies) there is cross-country trade data (due to being in the global input-output table) and it is possible to expand the model. While such an issue does not apply to the Iranian economy.

2.3. The framework of Gravity-FLQ method

To explain the Gravity-FLQ method, the multi-region table structure of this research is introduced. To this aim, first, the single-regional input-output table of each of the mentioned regions is prepared by the FLQ method using Eqs. (1) to (5). The structure of the single-regional table obtained according to the available data in the Iranian economy is presented in Table 1.

Table 1: Structure of single-regional input-output table used in research

Intermediate goods and services (internal)	Final demand (internal)	Output
Intermediate import from other regions in the country		
Intermediate import from foreign		
Value added		
Output		

As can be seen, the first area of the used single-regional table only includes internal intermediate exchanges of the same region. The final demand is the same. The Gravity-FLQ method emphasizes the intermediate imports from other regions of the country, which is the basis of distribution. Using the Gravity method, the amount of intermediate imports from other regions of the country is distributed[1]. Table 2 shows the structure of the multi-regional input-output table of the present research.

[1] In this section, the structure of Iran's single-regional input-output table is explained. Obviously, in other countries, the relationship between gravity can be used according to the available trade statistics between regions.

Table 2: The structure of the multi-regional table of the research

	Oil-rich regions	**Tehran**	**The rest of economies**	**Final demand (internal)**	**Output**
Oil-rich regions	Z^{OO}	Z^{OT}	Z^{OR}	Y^{O}	X^{O}
Tehran	Z^{TO}	Z^{TT}	Z^{TR}	Y^{T}	X^{T}
The rest of economies	Z^{RO}	Z^{RT}	Z^{RR}	Y^{R}	X^{R}
Intermediate import from [illegible]	M^{O}	M^{T}	M^{R}		
Value added	V^{O}	V^{T}	V^{R}		
Output	X^{O}	X^{T}	X^{R}		

In Table 2:

Z^{OO} is the square matrix of intra-regional intermediate exchanges within the oil-rich regions of Iran.

Z^{OT} is the matrix of inter-regional exchanges with the origin in oil-rich regions and the destination in Tehran.

Z^{OR} is the matrix of inter-regional exchanges with the origin in oil-rich regions and the destination in the rest of economies.

Z^{TO} is the matrix of inter-regional exchanges with the origin in Tehran and the destination in the oil-rich regions of Iran.

Z^{TT} is the square matrix of intra-regional intermediate exchanges with in Tehran.

Z^{TR} is the matrix of inter-regional exchanges with the origin in Tehran and the destination in the rest of economies.

Z^{RO} is the matrix of inter-regional exchanges with the origin of the rest of economies and the destination of the oil-rich regions of Iran.

Z^{RT} is the matrix of inter-regional exchanges with the origin of the rest of economies and the destination of Tehran.

Z^{RR} is the square matrix of intra-regional intermediate exchanges of the rest of economies.

Y^{O} is the column vector of domestic final demand for oil-rich regions, which includes household consumption, government expenditure, investment, and changes in inventory.

Y^{T} is the column vector of final domestic demand for Tehran that includes household consumption, government expenditure, investment, and changes in inventory.

Y^{R} is the column vector of the final domestic demand for the rest of economies, which includes household consumption, government expenditure, investment, and changes in inventory.

M^{O}, M^{T} and M^{R} are the import row vectors outside the oil-rich regions, Tehran, and the rest of economies, respectively.

V^{O}, V^{T}, and V^{R} are the value-added row vectors of oil-rich regions, Tehran, and the rest of economies, respectively.

X^{O}, X^{T}, and X^{R} are the output row vectors of oil-rich regions, Tehran, and the rest of economies, respectively.

The data in this section are as follows: Matrices Z^{OO} , Z^{TT} and Z^{RR} are calculated using FLQ method. Matrices Z^{OT} ,Z^{OR} ,Z^{TO}, Z^{TR}, Z^{RO} and Z^{RT} are calculated using GRAVITY method.

Matrix of intermediate import from foreign each region is obtained according to the ratio of region to country output in each sector.

Matrices Value added and Output are obtained from the Statistics Center of Iran.

Matrix of Final demand (internal) each region is obtained by reducing the sum of intermediate demands from the output.

The Gravity-FLQ method for constructing a multi-region input-output table is as follows:

A. Calculating the main diagonal matrices in the first area of the table, which represents the intra-regional exchange matrices. This area is estimated using the FLQ method.

B. Applying the model of Gravity according to Eq. (6): In this way, the intra-regional intermediate trade between the oil-rich regions (O) of the two regions of Tehran (T) and other national economies (R) is determined as Eqs. (7) and (8):

$$T_i^{TO} = \frac{X_i^T \times X_i^O}{X_{T+O}} * Q_i^{TO} \quad (7)$$

$$T_i^{RO} = \frac{X_i^R \times X_i^O}{X_{R+O}} * Q_i^{RO} \quad (8)$$

Q_i of each of Eqs. (7) and (8) is obtained using the output figures of the sectors and the distance between the centers of the mentioned regions. Therefore, the value of intermediate trade between the two oil-rich regions and Tehran (Eq.7) and the oil-rich region with other national economies (Eq. 8) is obtained separately and according to the equation of Gravity.

C.The relationship between the location quotient method and the Gravity model is established as follows. Intermediate imports calculated in the single-regional table (intermediate imports from other regions of the country) are distributed using the Gravity method. The intermediate import vector of oil-rich regions is distributed from the two regions of Tehran and other national economies according to Eqs. (9) and (10). So,

$$m_i^{TO} = \frac{T_i^{TO}}{T_i^{RO}+T_i^{TO}} * m_i^{O} \quad (9)$$

$$m_i^{RO} = \frac{T_i^{RO}}{T_i^{RO}+T_i^{TO}} * m_i^{O} \quad (10)$$

In Eqs. (9) and (10), m_i^{TO}, m_i^{RO}, and m_i^{O} are respectively the intra-regional intermediate imports of oil-rich regions from Tehran, the intra-regional intermediate imports of oil-rich regions from other national economies, and the value of intermediate imports of oil-rich regions. Eqs. (9) and (10) show the share of imports of oil-rich regions from the two regions of Tehran and other national economies. Since the values obtained from Eqs. (9) and (10) are row vectors, Eqs. (11) and (12) are used to construct the imports matrix (Ngo et al. 1986).

$$m^{TO} = A^{OO} * \text{diag}\, m^{TO} \quad (11)$$

$$m^{RO} = A^{OO} * \text{diag}\, m^{RO} \quad (12)$$

where $\text{diag}\, m^{TO}$, A^{OO}, and m^{TO} indicate the diagonal matrix of intermediate imports of oil-rich regions from Tehran, the technical coefficient matrix of oil-rich regions (importing region), and the intermediary import matrix of oil-rich regions from Tehran, respectively. Eq. (12) is the same as Eq. (11), except that it represents the import of oil-rich regions from other national economies. The inter-regional exchange matrices of Z^{TO} and Z^{RO} are calculated from Eqs. (13) and (14), respectively.

$$Z^{TO} = m^{TO} * \text{diag}\, X^{O} \quad (13)$$

$$Z^{RO} = m^{RO} * \text{diag}\, X^{O} \quad (14)$$

Where diag X^O is the output diagonal matrix of the oil-rich regions and the rest of the symbols have already been defined.

It is of note that the concept of Z is different from T in the above relations. T denotes the amount of trade that can exist between the two regions taking into account the three factors of production, demand, and distance, while Z shows the amount of trade between the two regions in addition to the above three factors, taking into account the sum of intermediate costs and intermediate demand.

2.4. Modeling multipliers coefficients and spillover & feedback effects in a multi-regional input-output table

In applied research, it is common to turn the input-output table into a model. For this purpose, the accounts in the table should divide into two categories: endogenous and exogenous. Intermediate exchanges (the first area of the table) are considered endogenous while final demand (the second area) and value-added (the third area) are considered exogenous. To derive a static multi-regional input-output model based on Table 2, we can write:

$$X^O = A^{OO}X^O + A^{OT}X^T + A^{OR}X^R + Y^O$$
$$X^T = A^{TO}X^O + A^{TT}X^T + A^{TR}X^R + Y^T \qquad (15)$$

$$X^R = A^{RO}X^O + A^{RT}X^T + A^{RR}X^R + Y^R$$

Using the assumption that the production function is linear, intra-regional technical coefficients and inter-regional trade coefficients can write as follows:

$$A^{OO} = Z^{OO}(X^O)^{-1} A^{TR} = Z^{TR}(X^R)^{-1} A^{OT} = Z^{OT}(X^T)^{-1}$$
$$A^{TT} = Z^{TT}(X^T)^{-1} A^{RT} = Z^{RT}(X^T)^{-1} A^{TO} = Z^{TO}(X^O)^{-1} \qquad (16)$$
$$A^{RR} = Z^{RR}(X^R)^{-1} A^{RO} = Z^{RO}(X^O)^{-1} A^{OR} = Z^{OR}(X^R)^{-1}$$

Where matrices A^{OO},A^{TT}, and A^{RR} show the intra-regional technical coefficients of oil-rich regions, Tehran, and other national economies, respectively. Also, matrices A^{TR}, A^{RT}, A^{OT}, A^{RO}, A^{TO}, and A^{OR} are inter-regional trade coefficients. If (16) is substituted in (15) and is written in the form of a matrix, (17) is obtained as follows.

$$\begin{pmatrix} X^O \\ X^T \\ X^R \end{pmatrix} = \begin{pmatrix} A^{OO} & A^{OT} & A^{OR} \\ A^{TO} & A^{TT} & A^{TR} \\ A^{RO} & A^{RT} & A^{RR} \end{pmatrix} \begin{pmatrix} X^O \\ X^T \\ X^R \end{pmatrix} + \begin{pmatrix} Y^O \\ Y^T \\ Y^R \end{pmatrix} \tag{17}$$

If Eq. (17) arranged according to the endogenous variables of the model, we will have:

$$\begin{pmatrix} X^O \\ X^T \\ X^R \end{pmatrix} = \left[\begin{pmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix} - \begin{pmatrix} A^{OO} & A^{OT} & A^{OR} \\ A^{TO} & A^{TT} & A^{TR} \\ A^{RO} & A^{RT} & A^{RR} \end{pmatrix} \right]^{-1} \begin{pmatrix} Y^O \\ Y^T \\ Y^R \end{pmatrix} \tag{18}$$
$$= \left[\begin{pmatrix} I - A^{OO} & -A^{OT} & -A^{OR} \\ -A^{TO} & I - A^{TT} & -A^{TR} \\ -A^{RO} & -A^{RT} & I - A^{RR} \end{pmatrix} \right]^{-1} \begin{pmatrix} Y^O \\ Y^T \\ Y^R \end{pmatrix}$$

Eq. (18) can be written in the matrix form as follows:

$$X = B * Y \tag{19}$$

Where X is the output vector, Y is the final demand vector, and B is Leontief multi-regional inverse matrix. In the main multi-regional model of relation (19), matrix B can be extended as follows:

$$B = \begin{pmatrix} B^{OO} & B^{OT} & B^{OR} \\ B^{TO} & B^{TT} & B^{TR} \\ B^{RO} & B^{RT} & B^{RR} \end{pmatrix} \tag{20}$$

Matrix (20) can be divided into three sub-matrices, each representing a specific effect.

$$B = \begin{pmatrix} B^{OO} & B^{OT} & B^{OR} \\ B^{TO} & B^{TT} & B^{TR} \\ B^{RO} & B^{RT} & B^{RR} \end{pmatrix}$$

$$= \begin{pmatrix} (I - A^{OO})^{-1} & 0 & 0 \\ 0 & (I - A^{TT})^{-1} & 0 \\ 0 & 0 & (I - A^{RR})^{-1} \end{pmatrix} + \begin{pmatrix} 0 & B^{OT} & B^{OR} \\ B^{TO} & 0 & B^{TR} \\ B^{RO} & B^{RT} & 0 \end{pmatrix}$$

$$+ \begin{pmatrix} B^{OO} - (I - A^{OO})^{-1} & 0 & 0 \\ 0 & B^{TT} - (I - A^{TT})^{-1} & 0 \\ 0 & 0 & B^{RR} - (I - A^{RR})^{-1} \end{pmatrix}$$

According to matrix analysis, three effects can be distinguished:

The first component shows the effect of multipliers coefficients of production. This effect is obtained from the column sum of the elements of the matrices, $(I - A^{OO})^{-1}(I - A^{TT})^{-1}$ and $(I - A^{RR})^{-1}$ and shows the direct and indirect effect of the change in the final demand of each region on the production of that region.

The second component describes the spillover effects. This effect is obtained from the column sum of the non-diagonal elements of matrix B, which shows the effect of the change in the final demand of one region on the production of other regions.

The third component includes feedback effects. This effect is obtained by summing the elements of the matrices $B^{OO} - (I - A^{OO})^{-1}$, $B^{TT} - (I - A^{TT})^{-1}$, and $B^{RR} - (I - A^{RR})^{-1}$. Feedback effects represent the recursive production effects caused by spillover effects. For example, according to the subject of research, the spillover effects caused by increased production in Tehran province on two other regions

(oil-rich regions and the rest of economies) increase production in Tehran province, which is called the feedback effect (Meng & Qu, 2007).

3. Empirical Results

3.1. Calculation of average multipliers coefficients, spillover and feedback effects

Table 3 shows the results of average multipliers coefficients, spillover effects, and feedback effects of the three study areas. This section was calculated using Eq. (20) and decomposition of the Leontief inverse matrix into three components of multipliers coefficients, spillover effects and feedback effects.

Table 3: Average multipliers coefficients, spillover & feedback effects of the three study regions

Region	Multipliers	Effect of spillover on oil-rich regions	Effect of spillover on the rest of economies	Effect of spillover on Tehran	Total of spillover effects	Feedback effects
Tehran	1.09	0.05	0.38	---	0.42	0.04
Oil-rich regions	1.21	---	0.22	0.05	0.27	0.02
The rest of economies	1.44	0.04	---	0.05	0.09	0.04

Source: Research Calculations

The results of Table 3 show that the highest multipliers coefficient is related to the region of the rest of economies. In other words, if the final demand in this region increases by one unit, the average production in the whole region will increase by 1.44 units. This result shows that intra-regional ties are stronger in the rest of economies than in the other two regions. The total effects of spillover in Tehran province are more than the other two regions because this region is smaller than the other two regions. The distribution of spillover effects is in line with the size and multipliers coefficient of the region. For example, the spillover effects of oil-rich regions have led to a higher increase in production in the rest of economies than in Tehran province (comparing 0.22 and 0.05). The region of the rest of economies during the period 2011-2016 accounted for about 50% of the production of the national economy. Meanwhile, the share of Tehran province was 20.5% during this period. Besides, the average multipliers coefficients in the region of the rest of economies are higher than in Tehran province. The same is true for the other two regions. The feedback effects obtained are less than the spillover values. This result is consistent with international studies such as Diazenbacher (2002). Considering the value of 0.02 related to the feedback effects of oil-rich regions, it can be said that with the increase of final demand in this region by one unit, production will increase by an average of 0.27 units in Tehran and the rest of economies. The increase in the above two regions, on average, causes 0.02-unit increase in oil-rich regions.

3.2. Calculation of Spillover effects of the first to third-ranked sectors in three study regions

To provide the desired report, the three sectors that have the most spillover effect are examined in Table 4. In Table 4, the rows represent the change in final demand in the region and the columns show the increase in production in the region (because of the change in final demand in another region). For example, the intersection of the final demand line of the rest of economies and the Tehran column reflects the spillover effects of the rest of economies on Tehran province.

The results of spillover effects between regions show that in the Tehran region and among the studied sectors, the industrial sector has the most spillover effects on the other two regions. With a 1-unit increase in the investment in the industrial sector of Tehran, the production of the industrial sector in oil-rich regions changes by 0.15 units and in the rest of economies by 0.90 units. The construction sector in Tehran province also has significant spillover effects. The reason for this result is the dependence of Tehran's economy on two other regions (especially the rest of economies) in this sector. Tehran province has the third-highest dependence in electricity, gas, and water production sector. In oil-rich regions, the construction sector has the most spillover effects. With a 1-unit increase in the investment in the construction sector of this region, production in Tehran province will increase by 0.10 units and in the rest of economies by 0.47 units. The spillover effects of the agricultural sector of this region are also significant. With a 1-unit increase in investment in the agricultural sector in oil-rich regions, production in the rest of economies will increase by 0.43 units. The greatest spillover effect of the rest of economies on Tehran is related to the information and communication services sector. The next ranks are for industry, financial services, and insurance. Meanwhile, the greatest spillover effect of the rest of economies on oil-rich regions is related to the industrial sector, followed by mining and construction. In general, the inter-regional spillover effects show that most of the activities related to information and communication services and financial and insurance services take place in Tehran province. Besides, the construction sector plays an important role in inter-regional spillover effects. Construction of roads, railways, urban utility projects, construction of electricity, gas, and water networks and related services are among the sub-activities of the construction sector that play an important role in inter-regional trade.

Table 4: Spillover effects of the first to third-ranked sectors in three study regions

Region	**Tehran**	**Oil-rich regions**	**The rest of economies**
Tehran	---	Mining:0.15 Industry: 0.15 Electricity, Gas, and Water Production: 0.13	Industry: 0.90 Construction: 0.71 Electricity, Gas, and Water Production: 0.57
Oil-rich regions	Construction: 0.10 Communication Services: 0.10 Financial and Insurance Services: 0.08	---	Construction: 0.47 Agriculture:0.43 Water Supply:0.40
The rest of economies	Communication Services: 0.17 Industry: 0.12 Financial and Insurance Services: 0.08	Industry: 0.14 Mining:0.13 Construction: 0.07	---

Source: Research Calculations

3.3. Calculation of feedback effects of the first to third-ranked sectors in three study regions

Table 5 shows the results of the feedback effects of the three studied sectors in the three regions. The calculations in this section are performed using the third component of Eq. (20).

Table 5: Feedback effects in three study regions

Region	Tehran	Oil-rich regions	The rest of economies
Tehran	Construction: 0.09 Mining:0.07 Communication Services: 0.07	---	---
Oil-rich regions	---	Industry: 0.05 Construction: 0.05 Water Supply:0.03	---
The rest of economies	---	---	Industry: 0.13 Construction: 0.08 Communication Services: 0.07

Source: Research Calculations

In Tehran province, the most feedback effect is related to the construction sector. Therefore, if the final demand of the construction sector in Tehran province increases by one unit, it will increase production in the rest of economies and in oil-rich regions. The increase in production in the other two regions leads to a 0.09-unit increase in the region itself.

4. Conclusion and Policy Implications

The main purpose of this study was to construct an input-output table of three important economic regions of Iran and to investigate their interaction with each other. For this purpose, first, three-region input-output table was prepared using FLQ-GRAVITY method. In this paper, due to the lack of interregional trade data in the Iranian economy, the basic model of gravity was used. However, in studies conducted in developed countries such as Krebs (2018) in Germany, the elasticity of each component of the gravity model is estimated with econometric models. The amount of trade between regions and their interdependence is measured using the effects of spillover and feedback in the form of a multi-regional input-output table. The results showed that Tehran (the capital of Iran) has the highest spillover effect among the three regions studied. In other words, it is most dependent on the production of the other two regions of Iran (oil-rich regions and the rest of economies). Among the economic sectors of Tehran province, industry has the most dependence. In addition, the results of the spillover effects show that Communication Services and Financial and Insurance Services are concentrated in Tehran. In general, the effects of spillover and feedback between regions on the Iranian economy are significant. Therefore, like the results of research by Zang (2017) and Li et al. (2018) in China, these effects should also be considered in policy-making in the Iranian economy.

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